

Flow Control: The Future

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The subject of flow control, particularly reactive flow control, is broadly introduced, leaving some of the details to other papers in this special volume of the *Journal of Aircraft*. The ability to manipulate a flowfield actively or passively to effect a desired change is of immense technological importance. In general, methods of control to achieve transition delay, separation postponement, lift enhancement, drag reduction, turbulence augmentation, and noise suppression are sought for both wall-bounded and free-shear flows. An attempt is made to present a unified view of the means by which different methods of control achieve a variety of end results. The important advances in the field of flow control that took place during the past few years are discussed. Spurred by the recent developments in chaos control, microfabrication and neural networks, reactive control of turbulent flows is now in the realm of the possible for future practical devices.

I. Introduction

THE ability to manipulate a flowfield actively or passively to effect a desired change is of immense technological importance, and this undoubtedly accounts for the subject being more hotly pursued by scientists and engineers than any other topic in fluid mechanics. The potential benefits of realizing efficient flow-control systems range from saving billions of dollars in annual fuel costs for land, air, and sea vehicles to achieving economically and environmentally more competitive industrial processes involving fluid flows. Methods of control to effect transition delay, separation postponement, lift enhancement, drag reduction, turbulence augmentation, and noise suppression are considered. Prandtl¹ pioneered the modern use of flow control in his epoch-making presentation to the Third International Congress of Mathematicians held at Heidelberg, Germany. In just eight pages, Prandtl introduced the boundary-layer theory, explained the mechanics of steady separation, opened the way for understanding the motion of real fluids, and described several experiments in which the boundary layer was controlled. He used active control of the boundary layer to show the great influence such control can exert on the flow pattern. Specifically, Prandtl used suction to delay boundary-layer separation from the surface of a cylinder.

Notwithstanding Prandtl's¹ success, aircraft designers in the three decades following his convincing demonstration were accepting lift and drag of airfoils as predestined characteristics with which no man could or should tamper.² This predicament changed mostly due to the German research in boundary-layer control pursued vigorously shortly before and during World War II. In the two decades following the war, extensive research on laminar flow control, where the boundary layer formed along the external surfaces of an aircraft is kept in the low-drag laminar state, was conducted in Europe and the United States, culminating in the successful flight test program of the X-21, where suction was used to delay transition on a swept wing up to a chord Reynolds number of 4.7×10^7 . The oil crisis of the early 1970s brought renewed interest in novel methods of flow control to reduce skin-friction drag even in turbulent boundary layers. In the 1990s, the need to reduce the emissions of greenhouse gases and to construct supermaneuverable fighter planes, faster/quieter underwater vehicles, and hypersonic transport aircraft, for example, the U.S. National Aerospace Plane, provides new challenges for researchers in the field of flow control.

Flow control, particularly reactive flow control, is broadly introduced in this paper, leaving some of the details to other papers in this volume, which will deal with specialized topics in flow control. Distributed flow control can greatly benefit from the availability of inexpensive microsensors and microactuators, hence the relevance of this topic to the present special issue of the *Journal of Aircraft*. In reactive control of turbulent flows, large arrays of minute sensors and actuators form feedback or feedforward control loops whose performance can be enhanced by employing optimized control algorithms and soft computing tools, hence the inclusion of this topic in the present paper.

Gad-el-Hak et al.³ and Gad-el-Hak⁴ provide an up-to-date overview of the subject of flow control. In this paper, following a description of the unifying principles of flow control, we focus on the concept of targeted control in which distributed arrays of microsensors and microactuators, connected in open or closed control loops, are used to target the coherent structures in turbulent flows to effect beneficial flow changes such as drag reduction, lift enhancement, noise suppression, etc.

II. Unifying Principles

A particular control strategy is chosen based on the kind of flow and the control goal to be achieved. Flow control goals are strongly, often adversely, interrelated, and there lies the challenge of making the tough compromises. There are several different ways for classifying control strategies to achieve a desired effect. Presence or lack of walls, Reynolds and Mach numbers, and the character of the flow instabilities are all important considerations for the type of control to be applied. All of these seemingly disparate issues are what places the field of flow control in a unified framework. In the following three subsections we will discuss the first three of those five issues. The reader is referred to Ref. 4 for a discussion of the latter two.

A. Control Goals and Their Interrelation

What does the engineer want to achieve when attempting to manipulate a particular flowfield? Typically the aim is at reducing the drag, at enhancing the lift, at augmenting the mixing of mass, momentum, or energy, at suppressing the flow-induced noise, or at a combination thereof. To achieve any of these useful end results, for either free-shear or wall-bounded flows, transition from laminar to turbulent flow may have to be either delayed or advanced, flow separation may have to be either prevented or provoked, and finally turbulence levels may have to be either suppressed or enhanced. All of those engineering goals and the corresponding flow changes intended to effect them are schematically shown in Fig. 1. None of that is particularly difficult if taken in isolation, but the challenge is in achieving a goal using a simple device, inexpensive to build as well as to operate, and, most important, has minimum side effects.

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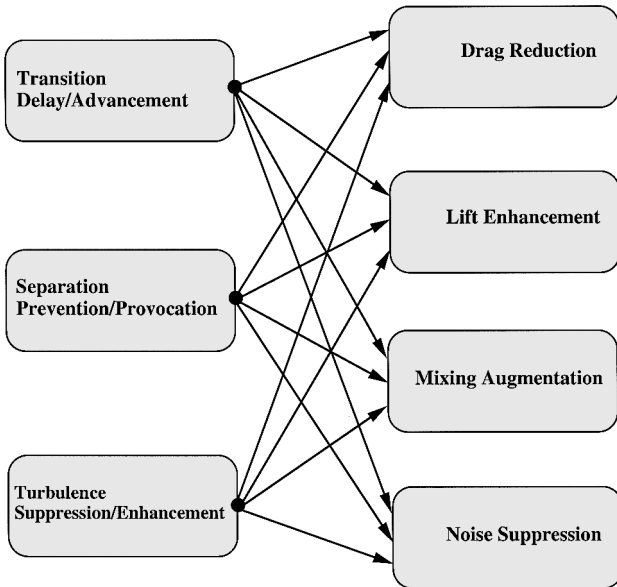


Fig. 1 Engineering goals and corresponding flow changes.

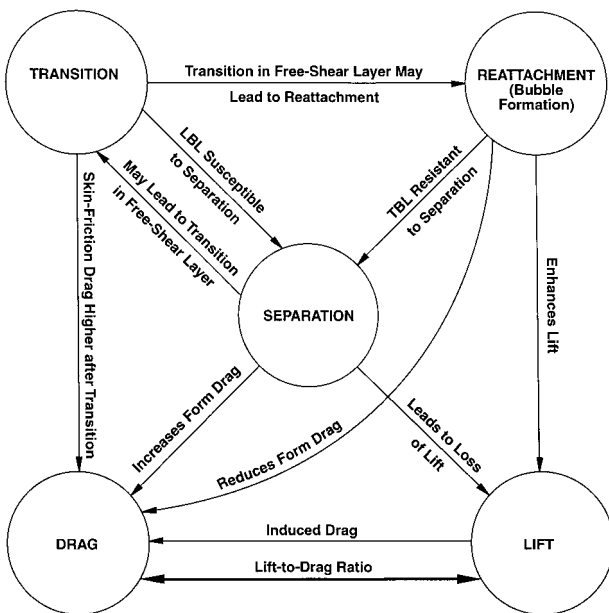


Fig. 2 Interrelation between flow control goals.

For this latter hurdle, the interrelation between control goals must be elaborated, and this is what is attempted next.

Consider the technologically very important boundary layers. An external wall-bounded flow, such as that developing on the exterior surfaces of an aircraft or a submarine, can be manipulated to achieve transition delay, separation postponement, lift increase, skin-friction and pressure drag reduction, turbulence augmentation, heat transfer enhancement, or noise suppression. (Note that pressure drag includes contributions from flow separation, displacement effects, induced drag, wave drag, and, for time-dependent motion of a body through a fluid, virtual mass.) These objectives are not necessarily mutually exclusive. The schematic in Fig. 2 is a partial representation of the interrelation between one control goal and another. To focus the discussion further, consider the flow developing on a lifting surface such as an aircraft wing. If the boundary layer becomes turbulent, its resistance to separation is enhanced and more lift can be obtained at increased incidence. On the other hand, the skin-friction drag for a laminar boundary layer can be as much as an order of magnitude less than that for a turbulent one. If transition is delayed, lower skin friction as well as lower flow-induced noise are achieved. However, the laminar boundary layer can only sup-

port very small adverse pressure gradient without separation and subsequent loss of lift and increase in form drag occur. Once the laminar boundary layer separates, a free-shear layer forms, and for moderate Reynolds numbers transition to turbulence takes place. Increased entrainment of high-speed fluid due to the turbulent mixing may result in reattachment of the separated region and formation of a laminar separation bubble. At higher incidence, the bubble breaks down either separating completely or forming a longer bubble. In either case, the form drag increases and the lift-curve's slope decreases. The ultimate goal of all of this is to improve the airfoil's performance by increasing the lift-to-drag ratio. However, induced drag is caused by the lift generated on a lifting surface with a finite span. Moreover, more lift is generated at higher incidence, but form drag also increases at these angles.

Potential conflicts arise as one tries to achieve a particular control goal only to affect adversely another goal. An ideal method of control that is simple, inexpensive to build and operate, and does not have any tradeoffs does not exist, and the skilled engineer has to make continuous compromises to achieve a particular design goal.

B. Classification Schemes

There are different classification schemes for flow control methods. One is to consider whether the technique is applied at the wall or away from it. Surface parameters that can influence the flow include roughness, shape, curvature, rigid-wall motion, compliance, temperature, and porosity. Heating and cooling of the surface can influence the flow via the resulting viscosity and density gradients. Mass transfer can take place through a porous wall or a wall with slots. Suction and injection of primary fluid can have significant effects on the flowfield, influencing particularly the shape of the velocity profile near the wall and thus the boundary layer susceptibility to transition and separation. Different additives, such as polymers, surfactants, microbubbles, droplets, particles, dust, or fibers, can also be injected through the surface in water or air wall-bounded flows. Control devices located away from the surface can also be beneficial. Large-eddy breakup devices (LEBU; also called outer-layer devices, OLD), acoustic waves bombarding a shear layer from outside, additives introduced in the middle of a shear layer, manipulation of freestream turbulence levels and spectra, gust, and magneto- and electrohydrodynamic body forces are examples of flow control strategies applied away from the wall.

A second scheme for classifying flow control methods considers energy expenditure and the control loop involved. As shown in Fig. 3, a control device can be passive, requiring no auxiliary

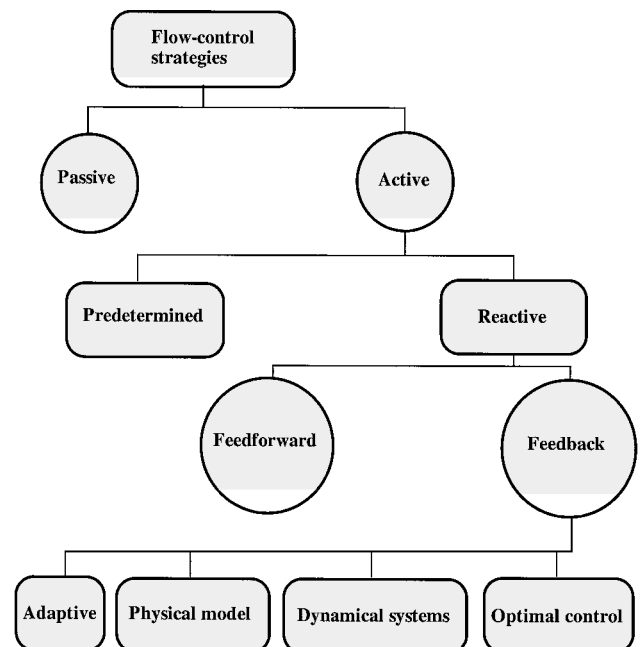


Fig. 3 Classification of flow control strategies.

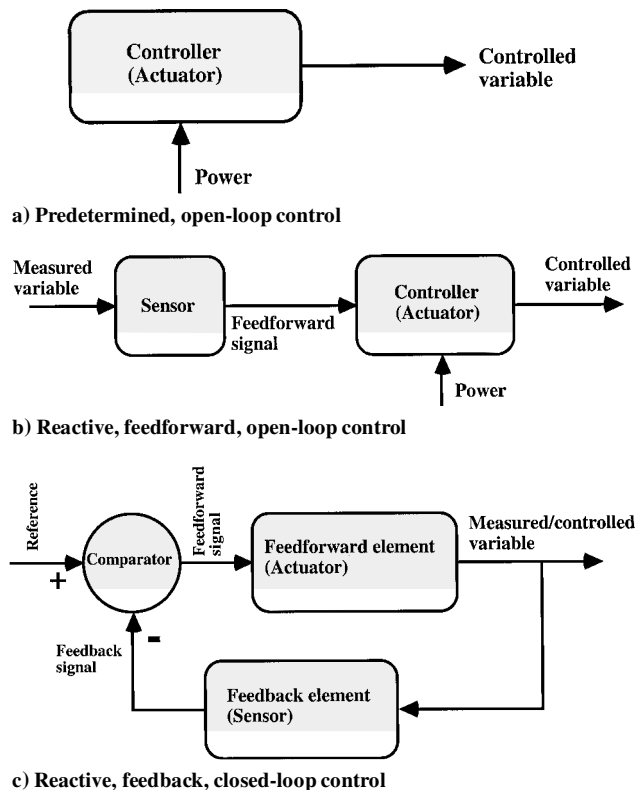


Fig. 4 Different control loops for active flow control.

power and no control loop, or active, requiring energy expenditure. As for the action of passive devices, some prefer to use the term flow management rather than flow control,⁵ reserving the latter terminology for dynamic processes. Active control requires a control loop and is further divided into predetermined or reactive. Predetermined control includes the application of steady or unsteady energy input without regard to the particular state of the flow. The control loop in this case is open, as shown in Fig. 4a, and no sensors are required. Because no sensed information is being fed forward, this open control loop is not a feedforward one. This subtle point is often confused in the literature, blurring predetermined control with reactive, feedforward control. Reactive control is a special class of active control where the control input is continuously adjusted based on measurements of some kind. The control loop in this case can either be an open, feedforward one (Fig. 4b) or a closed, feedback loop (Fig. 4c). Classical control theory deals, for the most part, with reactive control.

The distinction between feedforward and feedback is particularly important when dealing with the control of flow structures that convect over stationary sensors and actuators. In feedforward control, the measured variable and the controlled variable differ. For example, the pressure or velocity can be sensed at an upstream location, and the resulting signal is used together with an appropriate control law to trigger an actuator, which in turn influences the velocity at a downstream position. Feedback control, on the other hand, necessitates that the controlled variable be measured, fed back, and compared with a reference input. Reactive feedback control is further classified into four categories: adaptive, physical model based, dynamical-systems based, and optimal control.⁶

A yet another classification scheme is to consider whether the control technique directly modifies the shape of the instantaneous/mean velocity profile or selectively influence the small dissipative eddies. An inspection of the Navier-Stokes equations written at the surface⁴ indicates that the spanwise and streamwise vorticity fluxes at the wall can be changed, either instantaneously or in the mean, via wall motion/compliance, suction/injection, streamwise or spanwise pressure gradient (respectively), normal viscosity gradient, or a suitable streamwise or spanwise body force. These vorticity fluxes deter-

mine the fullness of the corresponding velocity profiles. For example, suction (or downward wall motion), favorable pressure gradient or lower wall viscosity results in vorticity flux away from the wall, making the surface a source of spanwise and streamwise vorticity. The corresponding fuller velocity profiles have negative curvature at the wall and are more resistant to transition and to separation but are associated with higher skin-friction drag. Conversely, an inflectional velocity profile can be produced by injection (or upward wall motion), adverse pressure gradient or higher wall viscosity. Such a profile is more susceptible to transition and to separation and is associated with lower, even negative, skin friction. Note that many techniques are available to effect a wall viscosity gradient, for example, surface heating/cooling, film boiling, cavitation, sublimation, chemical reaction, wall injection of lower/higher viscosity fluid, and the presence of shear thinning/thickening additive.

Flow control devices can alternatively target certain scales of motion rather than globally changing the velocity profile. Polymers, riblets, and LEBUs, for example, appear to damp selectively only the small dissipative eddies in turbulent wall-bounded flows. These eddies are responsible for the (instantaneous) inflectional profile and the secondary instability in the buffer zone, and their suppression leads to increased scales, a delay in the reduction of the (mean) velocity-profile slope, and consequent thickening of the wall region. In the buffer zone, the scales of the dissipative and energy containing eddies are roughly the same, and hence, the energy containing eddies will also be suppressed resulting in reduced Reynolds stress production, momentum transport, and skin friction.

C. Free-Shear and Wall-Bounded Flows

Free-shear flows, such as jets, wakes, and mixing layers, are characterized by inflectional mean-velocity profiles and are, therefore, susceptible to inviscid instabilities. Viscosity is only a damping influence in this case, and the prime instability mechanism is vortical induction. Control goals for such flows include transition delay/advancement, mixing enhancement, and noise suppression. External and internal wall-bounded flows, such as boundary layers and channel flows, can, too, have inflectional velocity profiles, but, in the absence of adverse pressure gradient and similar effects, are characterized by noninflectional profiles, and viscous instabilities are then to be considered. These kinds of viscosity-dominated wall-bounded flows are intrinsically stable and, therefore, are generally more difficult to control. Free-shear flows and separated boundary layers, on the other hand, are intrinsically unstable and lend themselves more readily to manipulation.

Free-shear flows originate from some kind of surface upstream, be it a nozzle, a moving body, or a splitter plate. Flow control devices can, therefore, be placed on the corresponding walls, albeit far from the fully developed regions. Examples of such control include changing of the geometry of a jet exit from circular to elliptic,⁷ using periodic suction/injection in the lee side of a blunt body to affect its wake,⁸ and vibrating the splitter plate of a mixing layer.⁹ These and other techniques are extensively reviewed by Fiedler and Fernholz,⁵ who offer a comprehensive list of appropriate references, and more recently by Gutmark et al.,¹⁰ Viswanath,¹¹ and Gutmark and Grinstein.¹²

III. Control of Turbulence

For the rest of this paper, we focus on reactive flow control specifically targeting the coherent structures in turbulent flows. Numerous methods of flow control have already been successfully implemented in practical engineering devices. Delaying laminar-to-turbulence transition to reasonable Reynolds numbers and preventing separation can readily be accomplished using a myriad of passive and predetermined active control strategies. Such classical techniques have been reviewed by, among others, Bushnell,^{13,14} Wilkinson et al.,¹⁵ Bushnell and McGinley,¹⁶ Gad-el-Hak,^{4,17} Bushnell and Hefner,¹⁸ Fiedler and Fernholz,⁵ Gad-el-Hak and Bushnell,¹⁹ Barnwell and Hussaini,²⁰ Viswanath,¹¹ and Joslin et al.²¹ However, very few of the classical strategies are effective in controlling free-shear or wall-bounded turbulent flows. Serious limitations exist for some familiar control techniques when applied to

certain turbulent flow situations. For example, in attempting to reduce the skin-friction drag of a body having a turbulent boundary layer using global suction, the penalty associated with the control device often exceeds the saving derived from its use. What is needed is a way to reduce this penalty to achieve a more efficient control.

Flow control is most effective when applied near the transition or separation points; in other words, near the critical flow regimes where flow instabilities magnify quickly. Therefore, delaying/advancing laminar-to-turbulence transition and preventing/provoking separation are relatively easier tasks to accomplish. To reduce the skin-friction drag in a nonseparating turbulent boundary layer, where the mean flow is quite stable, is a more challenging problem. However, even a modest reduction in the fluid resistance to the motion of, for example, the worldwide commercial airplane fleet is translated into fuel savings estimated to be in the billions of dollars. Newer ideas for turbulent flow control focus on the direct onslaught on coherent structures via reactive control strategies that utilize large arrays of microsensors and microactuators.

In the remaining sections of this paper, we advance possible scenarios by which viable control strategies of turbulent flows could be realized. As will be argued in the following sections, future systems for control of turbulent flows in general and turbulent boundary layers in particular could greatly benefit from the merging of the science of chaos control, the technology of microfabrication, and the newest computational tools collectively termed soft computing. Control of chaotic, nonlinear dynamic systems has been demonstrated theoretically as well as experimentally, even for multi-degree-of-freedom systems. Microfabrication is an emerging technology that has the potential for producing inexpensive, programmable sensor/actuator chips that have dimensions of the order of a few micrometers. Soft computing tools include neural networks, fuzzy logic and genetic algorithms, and are now more advanced as well as more widely used as compared to just few years ago. These tools could be very useful in constructing effective adaptive controllers.

Such futuristic systems are envisaged as consisting of a large number of intelligent, interactive, microfabricated wall sensors and actuators arranged in a checkerboard pattern and targeted toward specific organized structures that occur quasi-randomly within a turbulent flow. Sensors detect oncoming coherent structures, and adaptive controllers process the sensors information and provide control signals to the actuators, which in turn attempt to modulate favorably the quasi-periodic events. Finite number of wall sensors perceive only partial information about the entire flowfield. However, a low-dimensional dynamic model of the near-wall region used in a Kalman filter can make the most of the partial information from the sensors. Conceptually all of that is not too difficult, but in practice the complexity of such a control system is daunting and much research and development work still remains.

The following discussion is organized into seven sections. A particular example of a classical control system, suction, is described in the following section. This will serve as a prelude to introducing the selective suction concept. The characteristic lengths and sensor requirements of turbulent flows are then discussed in the subsequent section. This is followed by a description of reactive flow control and the selective suction concept. The number, size, frequency and energy consumption of the sensor/actuator units required to tame the turbulence on a full-scale air or water vehicle are estimated in therein. The emerging areas of chaos control and soft computing, particularly as they relate to reactive control strategies, are then briefly discussed in the two subsequent sections. This is followed by a discussion of the specific use of microelectromechanical systems (MEMS) devices for reactive flow control. Finally, brief concluding remarks are given in the last section.

IV. Suction

To set the stage for introducing the concept of targeted or selective control, in this section global control as applied to wall-bounded flows is discussed. A viscous fluid that is initially irrotational will acquire vorticity when an obstacle is passed through the fluid. This vorticity controls the nature and structure of the boundary-layer flow in the vicinity of the obstacle. For an incompressible, wall-

bounded flow, the flux of spanwise or streamwise vorticity at the wall, and hence whether the surface is a sink or a source of vorticity, is affected by the wall motion, for example, in the case of a compliant coating, transpiration (suction or injection), streamwise or spanwise pressure gradient, wall curvature, normal viscosity gradient near the wall (caused by, for example, heating/cooling of the wall or introduction of a shear-thinning/shear thickening additive into the boundary layer), and body forces (such as electromagnetic ones in a conducting fluid). These alterations separately or collectively control the shape of the instantaneous as well as the mean velocity profiles, which in turn determine the skin friction at the wall, the boundary-layer ability to resist transition and separation, and the intensity of turbulence and its structure.

For illustration purposes, global wall suction as a generic control tool is focused on. The arguments presented here and in subsequent sections are equally valid for other global control techniques, such as geometry modification (body shaping), surface heating/cooling, electromagnetic control, etc. Transpiration provides a good example of a single control technique that is used to achieve a variety of goals. Suction leads to a fuller velocity profile (vorticity flux away from the wall) and can, therefore, be employed to delay laminar-to-turbulence transition, postpone separation, achieve an asymptotic turbulent boundary layer (i.e., one having constant momentum thickness), or relaminarize an already turbulent flow. Unfortunately, global suction cannot be used to reduce the skin-friction drag in a turbulent boundary layer. The suction rate required to inhibit boundary-layer growth is too high to effect a net drag reduction. This is a good illustration of a situation where the penalty associated with a control device might exceed the saving derived from its use.

Small amounts of fluid withdrawn from the near-wall region of a boundary layer change the curvature of the velocity profile at the wall and can dramatically alter the stability characteristics of the flow. Concurrently, suction inhibits the growth of the boundary layer, so that the critical Reynolds number based on thickness may never be reached. Although laminar flow can be maintained to extremely high Reynolds numbers provided that enough fluid is sucked away, the goal is to accomplish transition delay with the minimum suction flow rate. This will reduce not only the power necessary to drive the suction pump, but also the momentum loss due to the additional freestream fluid entrained into the boundary layer as a result of withdrawing fluid from the wall. That momentum loss is, of course, manifested as an increase in the skin-friction drag.

The case of uniform suction from a flat plate at zero incidence is an exact solution of the Navier-Stokes equation. The asymptotic velocity profile in the viscous region is exponential and has a negative curvature at the wall. The displacement thickness has the constant value $\delta^* = \nu/|v_w|$, where ν is the kinematic viscosity and $|v_w|$ is the absolute value of the normal velocity at the wall. In this case, the familiar von Kármán integral equation is $C_f = 2C_q$. Bussmann and Münz²² computed the critical Reynolds number for the asymptotic suction profile to be $Re_{\delta^*} \equiv U_\infty \delta^*/\nu = 70,000$. From the given value of δ^* , the flow is stable to all small disturbances if $C_q \equiv |v_w|/U_\infty > 1.4 \times 10^{-5}$. The amplification rate of unstable disturbances for the asymptotic profile is an order of magnitude less than that for the Blasius boundary layer.²³ This treatment ignores the development distance from the leading edge needed to reach the asymptotic state. When this is included into the computation, a higher $C_q = 1.18 \times 10^{-4}$ is required to ensure stability.^{24,25}

In a turbulent wall-bounded flow, the results of Eléna^{26,27} and Antonia et al.²⁸ indicate that suction causes an appreciable stabilization of the low-speed streaks in the near-wall region. The maximum turbulence level at $y^+ \approx 13$ drops from 15% to 12% as C_q varies from 0 to 0.003. More dramatically, the tangential Reynolds stress near the wall drops by a factor of 2 for the same variation of C_q . The dissipation length scale near the wall increases by 40% and the integral length scale by 25% with the suction.

The suction rate necessary for establishing an asymptotic turbulent boundary layer independent of streamwise coordinate, that is, $d\delta_\theta/dx = 0$, is much lower than the rate required for relaminarization, $C_q \approx 0.01$, but is still not low enough to yield net drag reduction. For Reynolds number based on distance from leading edge

$Re_x = \mathcal{O}[10^6]$, Favre et al.,²⁹ Rotta,³⁰ and Verollet et al.,³¹ among others, report an asymptotic suction coefficient of $C_q \approx 0.003$. For a zero-pressure-gradient boundary layer on a flat plate, the corresponding skin-friction coefficient is $C_f = 2C_q = 0.006$, indicating higher skin friction than if no suction was applied. To achieve a net skin-friction reduction with suction, the process must be further optimized. One way to accomplish that is to target the suction toward particular organized structures within the boundary layer and not to use it globally as in classical control schemes.

V. Sensor Requirements

For boundary layers, the wall unit has been used to estimate the smallest necessary size of a sensor for accurately resolving the smallest eddies. For instance, Keith et al.³² state that 10 wall units or less is a relevant sensor dimension for resolving small-scale pressure fluctuations. Measurements of fluctuating velocity gradients, essential for estimating the total dissipation rate in turbulent flows, are another challenging task. Gad-el-Hak and Bandyopadhyay³³ argue that turbulence measurements with probe lengths greater than the viscous sublayer thickness (about five wall units) are unreliable particularly near the surface. Many studies have been conducted on the spacing between sensors necessary to optimize the formed velocity gradients (see Aronson et al.³⁴ and references therein). A general conclusion from both experiments and direct numerical simulations is that a sensor spacing of 3–5 Kolmogorov lengths is recommended. When designing arrays for correlation measurements or for targeted control, the spacing between the coherent structures will be the determining factor. For example, when targeting the low-speed streaks in a turbulent boundary layer, several sensors must be situated along a lateral distance of 100 wall units, the average spanwise spacing between streaks. All of this requires quite small sensors, and many attempts have been made to meet these conditions with conventional sensor designs. However, despite conventional sensors such as hot wires being fabricated in the micrometer size range (for their diameter but not their length), they are usually handmade, difficult to handle, and too fragile, and here the MEMS technology has really opened a door for new applications.

It is clear that the spatial and temporal resolutions for any probe to be used to resolve high-Reynolds-number turbulent flows are extremely tight. For example, both the Kolmogorov scale and the viscous length scale change from few micrometers at the typical field Reynolds number, based on the momentum thickness, of 10^6 , to a couple of hundred micrometers at the typical laboratory Reynolds number of 10^3 . MEMS sensors for pressure, velocity, temperature and shear stress are at least one order of magnitude smaller than conventional sensors.^{35–38} Their small size improves both the spatial and temporal resolutions of the measurements, typically few micrometers and few microseconds, respectively. For example, a micro hot wire (called hot point) has very small thermal inertia, and the diaphragm of a micropressure-transducer has correspondingly fast dynamic response. Moreover, the microsensors' extreme miniaturization and low energy consumption make them ideal for monitoring the flow state without appreciably affecting it. Last, literally hundreds of microsensors can be fabricated on the same silicon chip at a reasonable cost, making them well suited for distributed measurements and control.

VI. Reactive Flow Control

A. Introductory Remarks

Targeted control implies sensing and reacting to particular quasi-periodic structures in a turbulent flow. For a boundary layer, the wall seems to be the logical place for such reactive control because of the relative ease of placing something in there, the sensitivity of the flow in general to surface perturbations, and the proximity and therefore accessibility to the dynamically all important near-wall coherent events. According to Wilkinson,³⁹ there are very few actual experiments that use embedded wall sensors to initiate a surface actuator response.^{40–43} This 10-year-old assessment is fast changing, however, with the introduction of microfabrication technology that has the potential for producing small, inexpensive, programmable sensor/actuator chips. Witness the more recent reactive control attempts

by Kwong and Dowling,⁴⁴ Reynolds,⁴⁵ Jacobs et al.,⁴⁶ Jacobson and Reynolds,^{47–51} Fan et al.,⁵² James et al.,⁵³ and Keefe.⁵⁴ Fan et al.,⁵² and Jacobson and Reynolds,^{47–51} even consider the use of self-learning neural networks for increased computational speeds and efficiency. Recent reviews of reactive flow control include those by Gad-el-Hak,^{55,56} Lumley,⁵⁷ McMichael,⁵⁸ Mehregany et al.,⁵⁹ Ho and Tai,³⁵ and Bushnell.⁶⁰

B. Targeted Flow Control

Successful techniques to reduce the skin friction in a turbulent flow, such as polymers, particles, or riblets, appear to act indirectly through local interaction with discrete turbulent structures, particularly small-scale eddies, within the flow. Common characteristics of all of these passive methods are increased losses in the near-wall region, thickening of the buffer layer, and lowered production of Reynolds shear stress.⁶¹ Active control strategies that act directly on the mean flow, such as suction or lowering of near-wall viscosity, also lead to inhibition of Reynolds stress. However, skin friction is increased when any of these velocity-profile modifiers is applied globally.

Could these seemingly inefficient techniques, for example, global suction, be used more sparingly and be optimized to reduce their associated penalty? It appears that the more successful drag-reducing methods, for example, polymers, act selectively on particular scales of motion and are thought to be associated with stabilization of the secondary instabilities. It is also clear that energy is wasted when suction or heating/cooling is used to suppress the turbulence throughout the boundary layer when the main interest is to affect a near-wall phenomenon. One ponders, what would become of wall turbulence if specific coherent structures are to be targeted, by the operator through a reactive control scheme, for modification? The myriad of organized structures present in all shear flows are instantaneously identifiable, quasi-periodic motions.^{62,63} Bursting events in wall-bounded flows, for example, are both intermittent and random in space as well as time. The random aspects of these events reduce the effectiveness of a predetermined active control strategy. If such structures are nonintrusively detected and altered, however, net performance gain might be achieved. It seems clear, however, that temporal phasing as well as spatial selectivity would be required to achieve proper control targeted toward random events.

A nonreactive version of the described idea is the selective suction technique, which combines suction to achieve an asymptotic turbulent boundary layer and longitudinal riblets to fix the location of low-speed streaks. Although far from indicating net drag reduction, the available results are encouraging, and further optimization is needed. When implemented via an array of reactive control loops, the selective suction method is potentially capable of skin-friction reduction that approaches 60%.

The genesis of the selective suction concept can be found in the papers by Gad-el-Hak and Blackwelder^{64,65} and the patent by Blackwelder and Gad-el-Hak.⁶⁶ These researchers suggest that one possible means of optimizing the suction rate is to be able to identify where a low-speed streak is presently located and to apply a small amount of suction under it. When the production of turbulence kinetic energy is assumed to be due to the instability of an inflectional $U(y)$ velocity profile, one needs to remove only enough fluid so that the inflectional nature of the profile is alleviated. An alternative technique that could conceivably reduce the Reynolds stress is to inject fluid selectively under the high-speed regions. The immediate effect of normal injection would be to decrease the viscous shear at the wall resulting in less drag. In addition, the velocity profiles in the spanwise direction, $U(z)$, would have a smaller shear, $\partial U / \partial z$, because the suction/injection would create a more uniform flow. Inasmuch as Swearingen and Blackwelder⁶⁷ and Blackwelder and Swearingen⁶⁸ have found that inflectional $U(z)$ profiles occur as often as inflection points are observed in $U(y)$ profiles, suction under the low-speed streaks and/or injection under the high-speed regions would decrease this shear and, hence, the resulting instability.

The combination of selective suction and injection is shown in Fig. 5. In Fig. 5a, the vortices are idealized by a periodic distribution in the spanwise direction. The instantaneous velocity profiles

without transpiration at constant y and z locations are shown by the dashed lines in Figs. 5b and 5c, respectively. Clearly, the $U(y_0, z)$ profile is inflectional, having two inflection points per wavelength. At z_1 and z_3 , an inflectional $U(y)$ profile is also evident. The same profiles with suction at z_1 and z_3 and injection at z_2 are shown by the solid lines. In all cases, the shear associated with the inflection points would have been reduced. Because the inflectional profiles are all inviscidly unstable with growth rates proportional to the shear, the resulting instabilities would be weakened by the suction/injection process.

The feasibility of the selective suction as a drag-reducing concept has been demonstrated by Gad-el-Hak and Blackwelder⁶⁵ and is indicated in Fig. 6. Low-speed streaks were artificially generated in a laminar boundary layer using three spanwise suction holes using the method proposed by Gad-el-Hak and Hussain,⁶⁹ and a hot-film probe was used to record the near-wall signature of the streaks. An open, feedforward control loop with a phase lag was used to activate a predetermined suction from a longitudinal slot located in

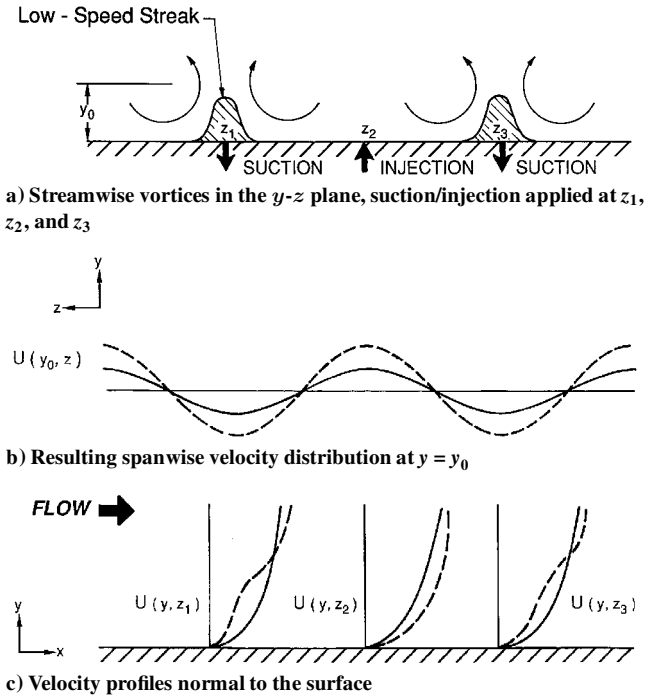


Fig. 5 Effects of suction/injection on velocity profiles; broken lines, reference profiles, and solid lines, profiles with transpiration applied.

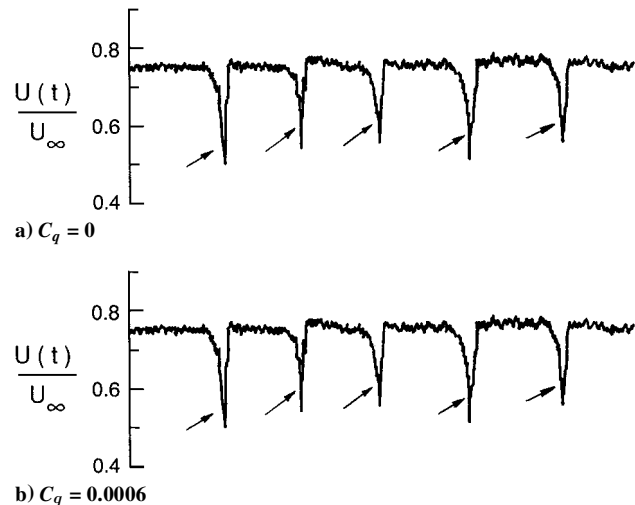


Fig. 6 Effects of suction from a streamwise slot on five artificially induced burstlike events in a laminar boundary layer (from Gad-el-Hak and Blackwelder⁶⁵).

between the spanwise holes and the downstream hot-film probe. An equivalent suction coefficient of $C_q = 0.0006$ was sufficient to eliminate the artificial events and prevent bursting. This rate is five times smaller than the asymptotic suction coefficient for a corresponding turbulent boundary layer. If this result is sustained in a naturally developing turbulent boundary layer, a skin-friction reduction of close to 60% would be attained.

Gad-el-Hak and Blackwelder⁶⁵ propose to combine suction with nonplanar surface modifications. Minute longitudinal roughness elements if properly spaced in the spanwise direction greatly reduce the spatial randomness of the low-speed streaks.⁷⁰ By withdrawing the streaks forming near the peaks of the roughness elements, less suction should be required to achieve an asymptotic boundary layer. Experiments by Wilkinson and Lazos⁷¹ and Wilkinson⁷² combine suction/blowing with thin-element riblets. Although no net drag reduction is yet attained in these experiments, their results indicate some advantage of combining suction with riblets as proposed by Gad-el-Hak and Blackwelder.^{64,65}

The recent numerical experiments of Choi et al.⁷³ also validate the concept of targeting suction/injection to specific near-wall events in a turbulent channel flow. Based on complete interior flow information and using the rather simple, heuristic control law proposed earlier by Gad-el-Hak and Blackwelder,⁶⁴ the Choi et al.⁷³ direct numerical simulations indicate a 20% net drag reduction accompanied by significant suppression of the near-wall structures and the Reynolds stress throughout the entire wall-bounded flow. When only wall information was used, a drag reduction of 6% was observed, which is a rather disappointing result considering that sensing and actuation took place at every grid point along the computational wall. In a practical implementation of this technique, even fewer wall sensors would perhaps be available, measuring only a small subset of the accessible information and, thus, requiring even more sophisticated control algorithms to achieve the same degree of success. Low-dimensional models of the near-wall flow and soft computing tools can help in constructing more effective control algorithms. These two topics will be revisited in Secs. VII and VIII, respectively.

Time sequences of the numerical flowfield of Choi et al.⁷³ indicate the presence of two distinct drag-reducing mechanisms when selective suction/injection is used. First is deterring the sweep motion, without modifying the primary streamwise vortices above the wall, and consequently moving the high-shear regions from the surface to the interior of the channel, thus directly reducing the skin friction. Second is changing the evolution of the wall vorticity layer by stabilizing and preventing lifting of the near-wall spanwise vorticity, thus suppressing a potential source of new streamwise vortices above the surface and interrupting a very important regeneration mechanism of turbulence.

Three modern developments have relevance to the issue at hand. First, the recently demonstrated ability to revert a chaotic system to a periodic one may provide optimal nonlinear control strategies for further reduction in the amount of suction (or the energy expenditure of any other active wall-modulation technique) needed to attain a given degree of flow stabilization. This is important because, as can be seen from the von Kármán integral momentum equation, net drag reduction achieved in a turbulent boundary layer increases as the suction coefficient decreases. Second, to remove selectively the randomly occurring low-speed streaks, for example, would ultimately require reactive control. In that case, an event is targeted, sensed and subsequently modulated. Microfabrication technology provides opportunities for practical implementation of the required large array of inexpensive, programmable sensor/actuator chips. Third, newly introduced soft computing tools include neural networks, fuzzy logic and genetic algorithms, and are now more advanced as well as more widely used as compared to just few years ago. These tools could be very useful in constructing effective adaptive controllers.

C. Reactive Feedback Control

As was schematically shown in Fig. 3, a control device can be passive, requiring no auxiliary power, or active, requiring energy expenditure. Active control is further divided into predetermined or

reactive. Predetermined control includes the application of steady or unsteady energy input without regard to the particular state of the flow. The control loop in this case is open, as was shown in Fig. 4a, and no sensors are required. Because no sensed information is being fed forward, this open control loop is not a feedforward one. Reactive control is a special class of active control where the control input is continuously adjusted based on measurements of some kind. The control loop in this case can either be an open, feedforward one (Fig. 4b) or a closed, feedback loop (Fig. 4c).

Moin and Bewley⁶ categorize reactive feedback control strategies by examining the extent to which they are based on the governing flow equations. Four categories are discerned: adaptive, physical model-based, dynamical-systems based, and optimal control (Fig. 3). Note that except for adaptive control, the other three categories of reactive feedback control can also be used in the feedforward mode or the combined feedforward-feedback mode. Also, in a convective environment such as that for a boundary layer, a controller would perhaps combine feedforward and feedback information and may include elements from each of the four classifications. Each of the four categories will be briefly described.

Adaptive schemes attempt to develop models and controllers via some learning algorithm without regard to the details of the flow physics. System identification is performed independently of the flow dynamics or the Navier–Stokes equations that govern this dynamics. An adaptive controller tries to optimize a specified performance index by providing a control signal to an actuator. To update its parameters, the controller, thus, requires feedback information relating to the effects of its control. The most recent innovation in adaptive flow control schemes involves the use of neural networks, which relate the sensor outputs to the actuator inputs through functions with variable coefficients and nonlinear, sigmoid saturation functions. The coefficients are updated using the so-called back-propagation algorithm, and complex control laws can be represented with a sufficient number of terms. Hand tuning is required, however, to achieve good convergence properties. The nonlinear adaptive technique has been used with different degrees of success by Fan et al.⁵² and by Jacobson and Reynolds^{48,50,51} to control, respectively, the transition process in laminar boundary layers and the bursting events in turbulent boundary layers.

Heuristic physical arguments can instead be used to establish effective control laws. That approach obviously will work only in situations in which the dominant physics are well understood. An example of this strategy is the active cancellation scheme, used by Gad-el-Hak and Blackwelder⁶⁵ in a physical experiment and by Choi et al.⁷³ in a numerical experiment, to reduce the drag by mitigating the effect of near-wall vortices. As mentioned earlier, the idea is to oppose the near-wall motion of the fluid, caused by the streamwise vortices, with an opposing wall control, thus lifting the high-shear region away from the surface and interrupting the turbulence regeneration mechanism.

Nonlinear dynamical systems theory allows turbulence to be decomposed into a small number of representative modes whose dynamics are examined to determine the best control law. The task is to stabilize the attractors of a low-dimensional approximation of a turbulent chaotic system. The best known strategy is the Ott–Grebogi–Yorke (OGY)^{74,75} method which, when applied to simpler, small-number of degrees-of-freedom systems, achieves stabilization with minute expenditure of energy. This and other chaos control strategies, especially as applied to the more complex turbulent flows, will be discussed later.

Finally, optimal control theory applied directly to the Navier–Stokes equations can, in principle, be used to minimize a cost function in the space of the control. This strategy provides perhaps the most rigorous theoretical framework for flow control. As compared to other reactive control strategies, optimal control applied to the full Navier–Stokes equations is also the most computer-time intensive. In this method, feedback control laws are derived systematically for the most efficient distribution of control effort to achieve a desired goal. Abergel and Temam⁷⁶ developed such optimal control theory for suppressing turbulence in a numerically simulated, two-dimensional Navier–Stokes flow, but their method requires an im-

practical full flowfield information. Choi et al.⁷⁷ developed a more practical, wall-information only, sub-optimal control strategy, which they applied to the one-dimensional stochastic Burgers equation. Later application of the suboptimal control theory to a numerically simulated turbulent channel flow has been reported by Moin and Bewley⁶ and Bewley et al.^{78,79} The recent book edited by Sriharan⁸⁰ provides eight articles that focus on the mathematical aspects of optimal control of viscous flows.

D. Required Characteristics

The randomness of the bursting events necessitates temporal phasing as well as spatial selectivity to effect selective control. Practical applications of methods targeted at controlling a particular turbulent structure to achieve a prescribed goal would, therefore, require implementing a large number of surface sensors/actuators together with appropriate control algorithms. That strategy for controlling wall-bounded turbulent flows has been advocated by, among others, Gad-el-Hak and Blackwelder,^{64,65} Lumley,^{57,81} Choi et al.,⁸² Reynolds,⁴⁵ Jacobson and Reynolds,^{47–51} Gad-el-Hak,^{4,55,56,83,84} Moin and Bewley,⁶ McMichael,⁵⁸ Mehregany et al.,⁵⁹ Blackwelder,⁸⁵ Delville et al.,⁸⁶ and Perrier.⁸⁷

It is instructive to estimate some representative characteristics of the required array of sensors/actuators. Consider a typical commercial aircraft cruising at a speed of $U_\infty = 300$ m/s and at an altitude of 10 km. The density and kinematic viscosity of air and the unit Reynolds number in this case are, respectively, $\rho = 0.4$ kg/m³, $\nu = 3 \times 10^{-5}$ m²/s, and $Re = 10^7$ /m. Assume further that the portion of fuselage to be controlled has turbulent boundary-layer characteristics that are identical to those for a zero-pressure-gradient flat plate at a distance of 1 m from the leading edge. In this case, the skin-friction coefficient and the friction velocity are, respectively, $C_f = 0.003$ and $u_\tau = 11.62$ m/s. (Note that the skin friction decreases as the distance from the leading edge increases. It is also strongly affected by such things as the externally imposed pressure gradient. Therefore, the estimates provided in here are for illustration purposes only.) At this location, one viscous wall unit is only $\nu/u_\tau = 2.6$ μ m. For the surface array of sensors/actuators to be hydraulically smooth, it should not protrude beyond the viscous sublayer, or $5\nu/u_\tau = 13$ μ m.

Wall-speed streaks are the most visible, reliable, and detectable indicators of the preburst turbulence production process. The detection criterion is simply low velocity near the wall, and the actuator response should be to accelerate (or to remove) the low-speed region before it breaks down. Local wall motion, tangential injection, suction, heating, or electromagnetic body force, all triggered on sensed wall-pressure or wall-shear stress, could be used to cause local acceleration of near-wall fluid.

The numerical experiments of Berkooz et al.⁸⁸ indicate that effective control of bursting pair of rolls may be achieved by using the equivalent of two wall-mounted shear sensors. If the goal is to stabilize or to eliminate all low-speed streaks in the boundary layer, a reasonable estimate for the spanwise and streamwise distances between individual elements of a checkerboard array is, respectively, 100 and 1000 wall units,³ or 260 μ m and 2600 μ m, for our particular example. Note that these are equal to, respectively, the average spanwise wavelength between two adjacent streaks and the average streamwise extent for a typical low-speed region. One can argue that those estimates are too conservative: Once a region is relaminarized, it would perhaps stay as such for quite a while as the flow convects downstream. The next row of sensors/actuators may, therefore, be relegated to a downstream location well beyond 1000 wall units. Relatively simple physical or numerical experiments could settle this issue. A reasonable size for each element is probably 1/10th of the spanwise separation, or 26 μ m. A (1 \times 1 m) portion of the surface would have to be covered with about $n = 1.5 \times 10^6$ elements. This is a colossal number, but the density of sensors/actuators could be considerably reduced if we moderate our goal of targeting every single bursting event (and also if less conservative assumptions are made).

It is well known that not every low-speed streak leads to a burst. On the average, a particular sensor would detect an incipient bursting

event every wall-unit interval of $P^+ = Pu_\tau^2/\nu = 250$, or $P = 56 \mu\text{s}$. The corresponding dimensionless and dimensional frequencies are $f^+ = 0.004$ and $f = 18 \text{ kHz}$, respectively. At different distances from the leading edge and in the presence of nonzero-pressure gradient, the sensors/actuators array would have different characteristics, but the corresponding numbers would still be in the same ballpark as estimated in here.

As a second example, consider an underwater vehicle moving at a speed of $U_\infty = 10 \text{ m/s}$. Despite the relatively low speed, the unit Reynolds number is still the same as estimated earlier for the air case, $Re = 10^7/\text{m}$, due to the much lower kinematic viscosity of water. At 1 m from the leading edge of an imaginary flat plate towed in water at the same speed, the friction velocity is only $u_\tau = 0.39 \text{ m/s}$, but the wall unit is still the same as in the aircraft example, $\nu/u_\tau = 2.6 \mu\text{m}$. The density of required sensors/actuators array is the same as computed for the aircraft example, $n = 1.5 \times 10^6 \text{ elements/m}^2$. The anticipated average frequency of sensing a bursting event is, however, much lower at $f = 600 \text{ Hz}$.

Similar calculations have been recently made by Gad-el-Hak,^{4,55,83,84} Reynolds,⁴⁵ and Wadsworth et al.⁸⁹ Their results agree closely with the estimates made here for typical field requirements. In either the airplane or the submarine case, the actuator's response need not be too large. As will be shown later, wall displacement on the order of 10 wall units ($26 \mu\text{m}$ in both examples), suction coefficient of about 0.0006, or surface cooling/heating on the order of $40^\circ\text{C}/2^\circ\text{C}$ (in the first/second example, respectively) should be sufficient to stabilize the turbulent flow.

As computed in the preceding two examples, both the required size for a sensor/actuator element and the average frequency at which an element would be activated are within the presently known capabilities of microfabrication technology. The number of elements needed per unit area is, however, alarmingly large. The unit cost of manufacturing a programmable sensor/actuator element would have to come down dramatically, perhaps matching the unit cost of a conventional transistor, before the idea advocated in here would become practical. (The transistor was invented in 1947. In the mid-1960s, a single transistor sold for around \$70. In 1997, Intel's Pentium II processor (microchip) contained 7.5×10^6 transistors and cost around \$500, which is less than \$0.00007 per transistor.)

An additional consideration to the size, amplitude, and frequency response is the energy consumed by each sensor/actuator element. Total energy consumption by the entire control system obviously has to be low enough to achieve net savings. Consider the following calculations for the aircraft example. At 1 m from the leading edge, the skin-friction drag to be reduced is approximately 54 N/m^2 . Engine power needed to overcome this retarding force per unit area is 16 kW/m^2 , or $10^4 \mu\text{W/sensor}$. If a 60% drag-reduction is achieved, this energy consumption is reduced to $4320 \mu\text{W/sensor}$. (A not-too-far-fetched goal according to the selective suction results presented earlier.) This number will increase by the amount of energy consumption of a sensor/actuator unit, but hopefully not back to the uncontrolled levels. The voltage across a sensor is typically in the range of $V = 0.1 - 1 \text{ V}$, and its resistance in the range of $R = 0.1 - 1 \text{ M}\Omega$. This means a power consumption by a typical sensor in the range of $P = V^2/R = 0.1 - 10 \mu\text{W}$, well below the anticipated power savings due to reduced drag.

For a single actuator in the form of a spring-loaded diaphragm with a spring constant of $k = 100 \text{ N/m}$ and oscillating up and down at the bursting frequency of $f = 18 \text{ kHz}$ with an amplitude of $y = 26 \mu\text{m}$, the power consumption is $P = (1/2)ky^2f = 600 \mu\text{W/actuator}$. If suction is used instead, $C_q = 0.0006$, and assuming a pressure difference of $\Delta p = 10^4 \text{ N/m}^2$ across the suction holes/slots, the corresponding power consumption for a single actuator is $P = C_q U_\infty \Delta p / n = 1200 \mu\text{W/actuator}$. It is clear then that when the power penalty for the sensor/actuator is added to the lower-level drag, a net saving is still achievable. The corresponding actuator power penalties for the submarine example are even smaller ($P = 20 \mu\text{W/actuator}$ for the wall motion actuator, and $P = 40 \mu\text{W/actuator}$ for the suction actuator), and larger savings are, therefore, possible.

VII. Chaos Control

In the theory of dynamical systems, the so-called butterfly effect denotes sensitive dependence of nonlinear differential equations on initial conditions, with phase-space solutions initially very close together separating exponentially. The solution of nonlinear dynamical systems of three or more degrees of freedom may be in the form of a strange attractor whose intrinsic structure contains a well-defined mechanism to produce a chaotic behavior without requiring random forcing. Chaotic behavior is complex, is aperiodic and, though deterministic, appears to be random.

A question arises naturally: Just as small disturbances can radically grow within a deterministic system to yield rich, unpredictable behavior, can minute adjustments to a system parameter be used to reverse the process and control, that is, regularize, the behavior of a chaotic system? Recently, that question was answered in the affirmative theoretically as well as experimentally, at least for system orbits that reside on low-dimensional strange attractors (see the review by Lindner and Ditto⁹⁰). Before describing such strategies for controlling chaotic systems, we first summarize the recent attempts to construct a low-dimensional dynamical systems representation of turbulent boundary layers. Such construction is a necessary first step to be able to use chaos control strategies for turbulent flows. Additionally, as argued by Lumley,⁵⁷ a low-dimensional dynamical model of the near-wall region used in a Kalman filter^{91,92} can make the most of the partial information assembled from a finite number of wall sensors. Such a filter minimizes in a least square sense the errors caused by incomplete information and, thus, globally optimizes the performance of the control system.

A. Nonlinear Dynamic Systems Theory

Boundary-layer turbulence is described by a set of nonlinear partial differential equations and is characterized by an infinite number of degrees of freedom. This makes it rather difficult to model the turbulence using a dynamical systems approximation. The notion that a complex, infinite-dimensional flow can be decomposed into several low-dimensional subunits is, however, a natural consequence of the realization that quasi-periodic coherent structures dominate the dynamics of seemingly random turbulent shear flows. This implies that low-dimensional, localized dynamics can exist in formally infinite-dimensional extended systems, such as open turbulent flows. Reducing the flow physics to finite-dimensional dynamical systems enables a study of its behavior through an examination of the fixed points and the topology of their stable and unstable manifolds. From the dynamical systems theory viewpoint, the meandering of low-speed streaks is interpreted as hovering of the flow state near an unstable fixed point in the low-dimensional state space. An intermittent event that produces high wall stress, a burst, is interpreted as a jump along a heteroclinic cycle to a different unstable fixed point that occurs when the state has wandered too far from the first unstable fixed point. Delaying this jump by holding the system near the first fixed point should lead to lower momentum transport in the wall region and, therefore, to lower skin-friction drag. Reactive control means sensing the current local state and through appropriate manipulation keeping the state close to a given unstable fixed point, thereby preventing further production of turbulence. Reducing the bursting frequency by, for example, 50%, may lead to a comparable reduction in skin-friction drag. For a jet, relaminarization may lead to a quiet flow and very significant noise reduction.

In one significant attempt, the proper orthogonal, or Karhunen-Loève, decomposition method has been used to extract a low-dimensional dynamical system from experimental data of the wall region (see Refs. 93 and 94). Aubry et al.⁹³ expanded the instantaneous velocity field of a turbulent boundary layer using experimentally determined eigenfunctions that are in the form of streamwise rolls. They expanded the Navier-Stokes equations using these optimally chosen, divergence-free, orthogonal functions, applied a Galerkin projection, and then truncated the infinite-dimensional representation to obtain a 10-dimensional set of ordinary differential equations. These equations represent the dynamical behavior of the rolls and are shown to exhibit a chaotic regime as well as an

intermittency due to a burstlike phenomenon. However, the Aubry et al. 10-mode dynamical system displays a regular intermittency, in contrast both to that in actual turbulence as well as to the chaotic intermittency encountered by Pomeau and Manneville⁹⁵ in which event durations are distributed stochastically. Nevertheless, the major conclusion of the Aubry et al.⁹³ study is that the bursts appear to be produced autonomously by the wall region even without turbulence, but are triggered by turbulent pressure signals from the outer layer. More recently, Berkooz et al.⁹⁶ generalized the class of wall-layer models developed by Aubry et al.⁹³ to permit uncoupled evolution of streamwise and cross-stream disturbances. The Berkooz et al.⁹⁶ results suggest that the intermittent events observed in the Aubry et al.⁹³ representation do not arise solely because of the effective closure assumption incorporated, but are rather rooted deeper in the dynamical phenomena of the wall region. Holmes et al.⁹⁷ detail the Cornell University research group attempts at describing turbulence as a low-dimensional dynamical system.

In addition to the reductionist viewpoint exemplified by the work of Aubry et al.⁹³ and Berkooz et al.,⁹⁶ attempts have been made to determine directly the dimension of the attractors underlying specific turbulent flows. Again, the central issue here is whether or not turbulent solutions to the infinite-dimensional Navier–Stokes equations can be asymptotically described by a finite number of degrees of freedom. Grappin and Léorat⁹⁸ computed the Lyapunov exponents and the attractor dimensions of two- and three-dimensional periodic turbulent flows without shear. They found that the number of degrees of freedom contained in the large scales establishes an upper bound for the dimension of the attractor. Deane and Sirovich^{99,100} numerically determined the number of dimensions needed to specify chaotic Rayleigh–Bénard convection over a moderate range of Rayleigh numbers Ra . They suggested that the intrinsic attractor dimension is $O[Ra^{2/3}]$.

The corresponding dimension in wall-bounded flows appears to be dauntingly high. Keefe et al.¹⁰¹ determined the dimension of the attractor underlying turbulent Poiseuille flows with spatially periodic boundary conditions. Using a coarse-grained numerical simulation, they computed a lower bound on the Lyapunov dimension of the attractor to be approximately 352 at a pressure-gradient Reynolds number of 3200. Keefe et al. argue that the attractor dimension in fully resolved turbulence is unlikely to be much larger than 780. This suggests that periodic turbulent shear flows are deterministic chaos and that a strange attractor does underlie solutions to the Navier–Stokes equations. Temporal unpredictability in the turbulent Poiseuille flow is, thus, due to the exponential spreading property of such attractors. Although finite, the computed dimension invalidates the notion that the global turbulence can be attributed to the interaction of a few degrees of freedom. Moreover, in a physical channel or boundary layer, the flow is not periodic and is open. The attractor dimension in such case is not known but is believed to be even higher than the estimate provided by Keefe et al. for the periodic (quasi-closed) flow.

In contrast to closed, absolutely unstable flows, such as Taylor–Couette systems, where the number of degrees of freedom can be small, local measurements in open, convectively unstable flows, such as boundary layers, do not express the global dynamics, and the attractor dimension in that case may inevitably be too large to be determined experimentally. According to the estimate provided by Keefe et al.,¹⁰¹ the colossal data required (about 10^D , where D is the attractor dimension) for measuring the dimension simply exceeds current computer capabilities. Turbulence near transition or near a wall is an exception to that bleak picture. In those special cases, a relatively small number of modes are excited, and the resulting simple turbulence can, therefore, be described by a dynamical system of a reasonable number of degrees of freedom.

B. Chaos Control

There is another question of greater relevance here. Given a dynamical system in the chaotic regime, is it possible to stabilize its behavior through some kind of active control? Although other alternatives have been devised (e.g., Refs. 102–105), the recent method proposed by workers at the University of Maryland^{74,75,106–111} promises to be a significant breakthrough. Comprehensive reviews

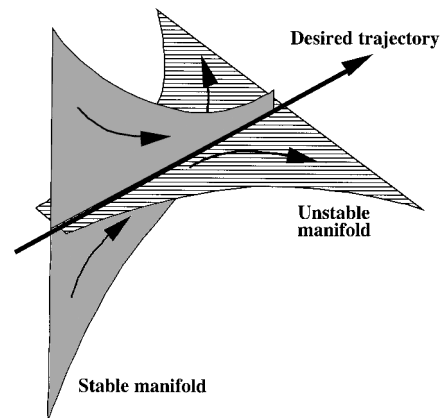


Fig. 7 OGY method for controlling chaos.

and bibliographies of the emerging field of chaos control can be found in the work by Shinbrot et al.,¹¹² Shinbrot,^{113–115} and Lindner and Ditto.⁹⁰

Ott et al.⁷⁴ demonstrated, through numerical experiments with the Henon map, that it is possible to stabilize a chaotic motion about any prechosen, unstable orbit through the use of relatively small perturbations. The procedure consists of applying minute time-dependent perturbations to one of the system parameters to control the chaotic system around one of its many unstable periodic orbits. In this context, targeting refers to the process whereby an arbitrary initial condition on a chaotic attractor is steered toward a prescribed point (target) on this attractor. The goal is to reach the target as quickly as possible using a sequence of small perturbations.¹¹⁶

The success of the Ott–Grebogi–Yorke (OGY) strategy for controlling chaos hinges on the fact that beneath the apparent unpredictability of a chaotic system lies an intricate but highly ordered structure. Left to its own recourse, such a system continually shifts from one periodic pattern to another, creating the appearance of randomness. An appropriately controlled system, on the other hand, is locked into one particular type of repeating motion. With such reactive control, the dynamical system becomes one with a stable behavior.

The OGY method can be simply illustrated by the schematic in Fig. 7. The state of the system is represented as the intersection of a stable manifold and an unstable one. The control is applied intermittently whenever the system departs from the stable manifold by a prescribed tolerance, otherwise the control is shut off. The control attempts to put the system back onto the stable manifold so that the state converges toward the desired trajectory. Unmodeled dynamics cause noise in the system and a tendency for the state to wander off in the unstable direction. The intermittent control prevents that, and the desired trajectory is achieved. This efficient control is not unlike trying to balance a ball in the center of a horse saddle.⁶ There is one stable direction (front/back) and one unstable direction (left/right). The restless horse is the unmodeled dynamics, intermittently causing the ball to move in the wrong direction. The OGY control needs only be applied, in the most direct manner possible, whenever the ball wanders off in the left/right direction.

The OGY method has been successfully applied in a relatively simple experiment in which reverse chaos was obtained in a parametrically driven, gravitationally buckled, amorphous magnetoelastic ribbon.^{117,118} Garfinkel et al.¹¹⁹ applied the same control strategy to stabilize drug-induced cardiac arrhythmias in sections of a rabbit ventricle. Other extensions, improvements, and applications of the OGY strategy include higher-dimensional targeting;^{120,121} controlling chaotic scattering in Hamiltonian, that is, nondissipative, area conservative, systems;^{122,123} synchronization of identical chaotic systems that govern communication, neural, or biological processes;¹²⁴ use of chaos to transmit information;^{125,126} control of transient chaos;¹²⁷ and taming spatio-temporal chaos using a sparse array of controllers.^{128–130}

In a more complex system, such as a turbulent boundary layer, there exist numerous interdependent modes and many stable as well

as unstable manifolds (directions). The flow can then be modeled as coherent structures plus a parameterized turbulent background. The proper orthogonal decomposition (POD) is used to model the coherent part because POD guarantees the minimum number of degrees of freedom for a given model accuracy. Factors that make turbulence control a challenging task are the potentially quite large perturbations caused by the unmodeled dynamics of the flow, the non stationary nature of the desired dynamics, and the complexity of the saddle shape describing the dynamics of the different modes. Nevertheless, the OGY control strategy has several advantages that are of special interest in the control of turbulence: 1) The mathematical model for the dynamic system need not be known. 2) Only small changes in the control parameter are required. 3) Noise can be tolerated (with appropriate penalty).

Recently, Keefe^{131,132} made a useful comparison between two nonlinear control strategies as applied to fluid problems. The OGY feedback method described earlier and the model-based control strategy originated by Hübner (for example, see Hübner and Lüscher¹⁰³ and Lüscher and Hübner¹³³), the H-method. Both novel control methods are essentially generalizations of the classical perturbation cancellation technique: Apply a prescribed forcing to subtract the undesired dynamics and impose the desired one. The OGY strategy exploits the sensitivity of chaotic systems to stabilize existing periodic orbits and steady states. Some feedback is needed to steer the trajectories toward the chosen fixed point, but the required control signal is minuscule. In contrast, Hübner's scheme (see Refs. 103 and 133) does not explicitly make use of the system sensitivity. It produces general control response (periodic or aperiodic) and needs little or no feedback, but its control inputs are generally large. The OGY strategy exploits the nonlinearity of a dynamic system; indeed the presence of a strange attractor and the extreme sensitivity of the dynamic system to initial conditions are essential to the success of this method. In contrast, the H-method works equally for both linear and nonlinear systems.

Keefe¹³¹ first examined numerically the two schemes as applied to fully-developed and transitional solutions of the Ginzburg–Landau equation, an evolution equation that governs the initially weakly nonlinear stages of transition in several flows and that possesses both transitional and fully-chaotic solutions. The Ginzburg–Landau equation has solutions that display either absolute or convective instabilities and is, thus, a reasonable model for both closed and open flows. Keefe's¹³¹ main conclusion is that control of nonlinear systems is best obtained by making maximum use possible of the underlying natural dynamics. If the goal dynamics is an unstable nonlinear solution of the equation and the flow is nearby at the instant control is applied, both methods perform reliably and at low-energy cost in reaching and maintaining this goal. Predictably, the performance of both control strategies degrades due to noise and the spatially discrete nature of realistic forcing.

Subsequently, Keefe¹³² extended the numerical experiment in an attempt to reduce the drag in a channel flow with spatially periodic boundary conditions. The OGY method reduces the skin friction to 60–80% of the uncontrolled value at a mass-flux Reynolds number of 4408. The H-method fails to achieve any drag reduction when starting from a fully turbulent initial condition but shows potential for suppressing or retarding laminar-to-turbulence transition. Keefe¹³¹ suggests that the H-strategy might be more appropriate for boundary layer control, whereas the OGY-method might best be used for channel flows.

It is also relevant to note here the work of Singer et al.¹³⁴ and Wang et al.¹³⁵ at the University of Pennsylvania, who devised a feedback control to stabilize (relaminarize) the naturally occurring chaotic oscillations of a toroidal thermal convection loop heated from below and cooled from above. Based on a simple mathematical model for the thermosyphon, Singer et al.¹³⁴ and Wang et al.¹³⁵ constructed a reactive control system that was used to alter significantly the flow characteristics inside the convection loop. Their linear control strategy, perhaps a special version of the OGY's chaos control method, consists simply of sensing the deviation of fluid temperatures from desired values at a number of locations inside the thermosyphon loop and then altering the wall heating either to suppress or to en-

hance such deviations. Wang et al. also suggested extending their theoretical and experimental method to more complex situations such as those involving Bénard convection (see Refs. 136 and 137). Hu and Bau¹³⁸ used a similar feedback control strategy to demonstrate that the critical Reynolds number for the loss of stability of planar Poiseuille flow can be significantly increased or decreased.

Other attempts to use low-dimensional dynamic systems representation for flow control include the work of Berkooz et al.,⁸⁸ Corke et al.,¹³⁹ and Collier et al.^{140,141} Berkooz et al.⁸⁸ applied techniques of modern control theory to estimate the phase-space location of dynamical models of the wall-layer coherent structures and used these estimates to control the model dynamics. Because discrete wall sensors provide incomplete knowledge of phase-space location, Berkooz et al.¹³⁹ maintain that a nonlinear observer, which incorporates past information and the equations of motion into the estimation procedure, is required. Using an extended Kalman filter, they achieved effective control of a bursting pair of rolls with the equivalent of two wall-mounted shear sensors. Corke et al.¹³⁹ used a low-dimensional dynamical system based on the proper orthogonal decomposition to guide control experiments for an axisymmetric jet. When the downstream velocity is sensed and an array of miniature speakers located at the lip of the jet is actuated, their feedback control succeeded in converting the near-field instabilities from spatial-convective to temporal-global. Collier et al.^{140,141} developed a feedback control strategy for strongly nonlinear dynamical systems, such as turbulent flows, subject to small random perturbations that kick the system intermittently from one saddle point to another along heteroclinic cycles. In essence, their approach is to use local, weakly nonlinear feedback control to keep a solution near a saddle point as long as possible, but then to let the natural, global nonlinear dynamics run its course when bursting (in a low-dimensional model) does occur. Though conceptually related to the OGY strategy, the Collier et al.⁵ method does not actually stabilize the state but merely holds the system near the desired point longer than it would otherwise stay.

Shinbrot and Ottino^{142,143} offer yet another strategy, presumably most suited for controlling coherent structures in area-preserving turbulent flows. Their geometric method exploits the premise that the dynamical mechanisms that produce the organized structures can be remarkably simple. By repeated stretching and folding of horseshoes that are present in chaotic systems, Shinbrot and Ottino have demonstrated numerically as well as experimentally the ability to create, destroy, and manipulate coherent structures in chaotic fluid systems. The key idea to create such structures is to intentionally place folds of horseshoes near low-order periodic points. In a dissipative dynamical system, volumes contract in state space and the collocation of a fold with a periodic point leads to an isolated region that contracts asymptotically to a point. Provided that the folding is done properly, it counteracts stretching. Shinbrot and Ottino¹⁴² applied the technique to three prototypical problems: a one-dimensional chaotic map, a two-dimensional one, and a chaotically advected fluid. Shinbrot^{114,115} and Shinbrot et al.¹¹⁰ provide recent reviews of the stretching/folding as well as other chaos control strategies.

VIII. Soft Computing

The term soft computing was coined by Lotfi Zadeh of the University of California, Berkeley, to describe several ingenious modes of computations that exploit tolerance for imprecision and uncertainty in complex systems to achieve tractability, robustness and low cost.^{144–147} The principle of complexity provides the impetus for soft computing: As the complexity of a system increases, the ability to predict its response diminishes until a threshold is reached beyond which precision and relevance become almost mutually exclusive.¹⁴⁸ In other words, precision and certainty carry a cost. By employing modes of reasoning—probabilistic reasoning—that are approximate rather than exact, soft computing can help in searching for globally optimal design or achieving effectual control while taking into account system uncertainties and risks.

Soft computing refers to a domain of computational intelligence that loosely lies in between purely numerical (hard) computing and purely symbolic computations. Alternatively, one can think about symbolic computations as a form of artificial intelligence lying in

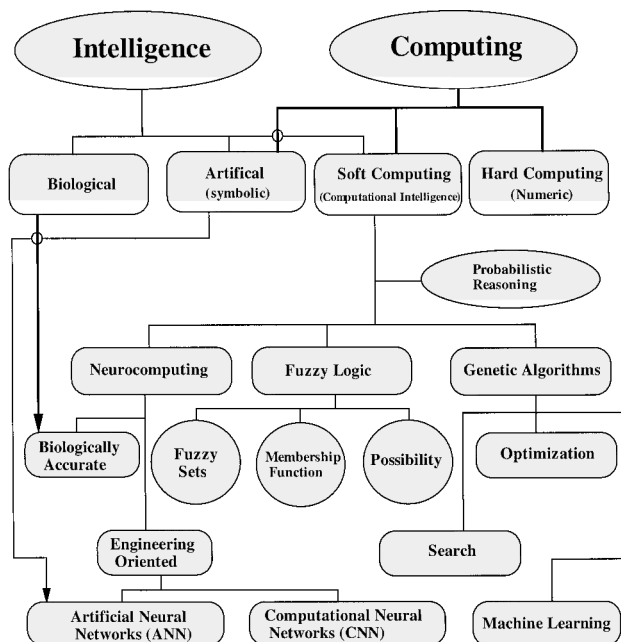


Fig. 8 Tools for soft computing.

between biological intelligence and computational intelligence (soft computing). The schematic in Fig. 8 shows the general idea. Artificial intelligence relies on symbolic information processing techniques and uses logic as representation and inference mechanisms. It attempts to approach the high level of human cognition. In contrast, soft computing is based on modeling low-level cognitive processes and strongly emphasizes modeling of uncertainty as well as learning. Computational intelligence mimics the ability of the human brain to employ modes of reasoning that are approximate. Soft computing provides a machinery for the numeric representation of the types of constructs developed in the symbolic artificial intelligence. The boundaries between these paradigms are of course fuzzy.

The principal constituents of soft computing are neurocomputing, fuzzy logic, and genetic algorithms, as shown in Fig. 8. These elements, together with probabilistic reasoning, can be combined in hybrid arrangements resulting in better systems in terms of parallelism, fault tolerance, adaptivity and uncertainty management. To the author's knowledge, only neurocomputing has been employed for fluid flow control, but the other tools of soft computing may be just as useful to construct powerful controllers and have in fact been used as such in other fields such as large-scale subway controllers and video cameras. A brief description of those three constituents follows.

Neurocomputing is inspired by the neurons of the human brain and how they work. Neural networks are information processing devices that can learn by adapting synaptic weights to changes in the surrounding environment; can handle imprecise, fuzzy, noisy and probabilistic information; and can generalize from known tasks (examples) to unknown ones. Actual engineering oriented hardware are termed artificial neural networks (ANN), whereas algorithms are called computational neural networks (CNN). The nonlinear, highly parallel networks can perform any of the following tasks: classification, pattern matching, optimization, control, and noise removal. As modeling and optimization tools, neural networks are particularly useful when good analytic models are either unknown or extremely complex.

An ANN consists of a large number of highly interconnected processing elements, essentially equations known as transfer functions, that are analogous to human neurons and are tied together with weighted connections that are analogous to human synapses. A processing unit takes weighted signals from other units, possibly combines them, and gives a numeric result. The behavior of neural networks—how they map input data—is influenced primarily by the transfer functions of the processing elements, how the transfer functions are interconnected, and the weights of those intercon-

nections. Learning typically occurs by example—through exposure to a set of input-output data—where the training algorithm adjusts the connection weights (synapses). These connection weights store the knowledge necessary to solve specific problems. As an example, it is now possible to use neural networks to sense (smell) odors in many different applications.¹⁴⁹ The electronic noses (e-noses) are on the verge of finding commercial applications in medical diagnostics, environmental monitoring, and the processing and quality control of foods. Neural networks as used in fluid flow control will be covered in the following subsection.

Fuzzy logic was introduced by Zadeh in 1965 as a mathematical tool to deal with uncertainty and imprecision. The book edited by Yager and Zadeh¹⁴⁴ is an excellent primer to the field. For computing and reasoning, general concepts (such as size) are implemented into a computer algorithm by using mostly words (such as small, medium or large). Fuzzy logic, therefore, provides a unique methodology for computing with words. Its rationalism is based on three mathematical concepts: fuzzy sets, membership function, and possibility. As dictated by a membership function, fuzzy sets allow a gradual transition from belonging to not belonging to a set. The concept of possibility provides a mechanism for interpreting factual statements involving fuzzy sets. Three processes are involved in solving a practical problem using fuzzy logic: fuzzification, analysis, and defuzzification. Given a complex, unsolvable problem in real space, those three steps involve enlarging the space and searching for a solution in the new superset, then specializing this solution to the original real constraints.

Genetic algorithms are search algorithms based loosely on the mechanics of natural selection and natural genetics. They combine survival of the fittest among string structures with structured yet randomized information exchange and are used for search, optimization and machine learning. For control, genetic algorithms aim at achieving minimum cost function and maximum performance measure while satisfying the problem constraints. Goldberg,¹⁵⁰ Davis,¹⁵¹ and Holland¹⁵² provide gentle introduction to the field.

In the Darwinian principle of natural selection, the fittest members of a species are favored to produce offspring. Even biologists cannot help but being awed by the complexity of life observed to evolve in the relatively short time suggested by the fossil records. A living being is an amalgam of characteristics determined by the (typically tens of thousands) genes in its chromosomes. Each gene may have several forms or alternatives called alleles, which produce differences in the set of characteristics associated with that gene. The chromosomes are, therefore, the organic devices through which the structure of a creature is encoded, and this living being is created partly through the process of decoding those chromosomes. Genes transmit hereditary characters and form specific parts of a self-perpetuated deoxyribonucleic acid in a cell nucleus. Natural selection is the link between the chromosomes and the performance of their decoded structures. Simply put, the process of natural selection causes those chromosomes that encode successful structures to reproduce more often than those that do not.

In an attempt to solve difficult problems, John H. Holland of the University of Michigan introduced in the early 1970s the fabricated version of the procedure of natural evolution. The candidate solutions to a problem are ranked by the genetic algorithm according to how well they satisfy a certain criterion, and the fittest members are the most favored to combine amongst themselves to form the next generation of the members of the species. Fitter members presumably produce even fitter offspring and, therefore, better solutions to the problem at hand. Solutions are represented by binary strings; each trial solution is coded as a vector called a chromosome. The elements of a chromosome are described as genes, and its varying values at specific positions are called alleles. Good solutions are selected for reproduction based on a fitness function using genetic recombination operators such as crossover and mutation. The main advantage of genetic algorithms is their global parallelism in which the search efforts to many regions of the search area are simultaneously allocated.

Genetic algorithms have been used for the control of different dynamical systems, as, for example, the optimization of robot

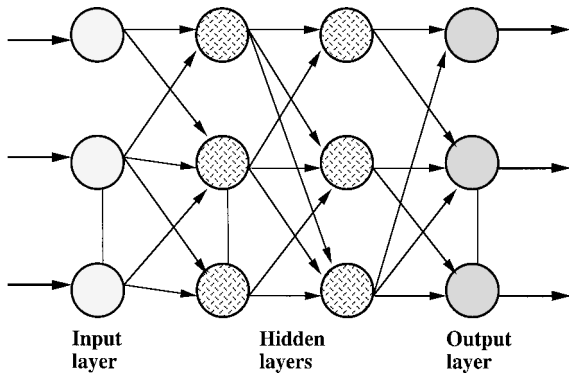


Fig. 9 Elements of a neural network.

trajectories. However, to the author's knowledge and at the time of writing this paper, reactive control of turbulent flows is yet to benefit from this powerful soft computing tool. In particular, when a finite number of sensors are used to gather information about the state of the flow, a genetic algorithm perhaps combined with a neural network can adapt and learn to use current information to eliminate the uncertainty created by insufficient a priori information.

Neural Networks for Flow Control

Biologically inspired neural networks are finding increased applications in many fields of science and technology. Modeling of complex dynamical systems, adaptive noise canceling in telephones and modems, bomb sniffers, mortgage-risk evaluators, sonar classifiers, and word recognizers are but a few of existing usage of neural nets. Nelson and Illingworth¹⁵³ provide a lucid introduction to the field, and Antsaklis¹⁵⁴ focuses on the use of neural nets for the control of complex dynamical systems. For flow control applications, neural networks offer the possibility of adaptive controllers that are simpler and potentially less sensitive to parameter variations as compared to conventional controllers. Moreover, if a colossal number of sensors and actuators is to be used, the massively parallel computational power of neural nets will surely be needed for real-time control.

The basic elements of a neural network are schematically shown in Fig. 9. Several inputs are connected to the nodes (neurons or processing elements) that form the input layer. There are one or more hidden layers, followed by an output layer. Note that the number of connections is higher than the total number of nodes. Both numbers are chosen based on the particular application and can be arbitrarily large for complex tasks. Simply put, the multitask—albeit simple—job of each processing element is to evaluate each of the input signals to that particular element, calculate the weighted sum of the combined inputs, compare that total to some threshold level, and finally determine what the output should be. The various weights are the adaptive coefficients, which vary dynamically as the network learns to perform its assigned task; some inputs are more important than others. The threshold, or transfer, function is generally nonlinear; the most common one being the continuous sigmoid, or S-shaped, curve, which approaches a minimum and maximum value at the asymptotes. If the sum of the weighted inputs is larger than the threshold value, the neuron generates a signal; otherwise, no signal is fired. Neural networks can operate in feedforward or feedback mode. (Note that this terminology refers to the direction of information through the network. When a neural net is used as a controller, the overall control loop is, however, a feedback, closed loop: The self-learning network dynamically updates its various parameters by comparing its output to a desired output, thus requiring feedback information relating to the effect of its control.) Complex systems, for which dynamic equations may not be known or may be too difficult to solve, can be modeled using neural nets.

For flow control, neural networks provide convenient, fast, nonlinear adaptive algorithms to relate sensor outputs to actuator inputs via variable-coefficient functions and nonlinear, sigmoid saturation functions. With no prior knowledge of the pertinent dynamics, a self-learning neural network develops a model for that dynamics

through observations of the applied control and sensed measurements. The network is by nature nonlinear and can, therefore, better handle nonlinear dynamical systems, a difficult task when classical (linear or weakly nonlinear) control strategies are attempted. The feedforward type of neural network acts as a nonlinear filter forming an output from a set of input data. The output can then be compared to some desired output, and the difference (error) is typically used in a back-propagation algorithm, which updates the network parameters.

The number of researchers using neural networks to control fluid flows is growing rapidly. In here, we provide only a small sample. Using a pretrained neural network, Fan et al.⁵² conducted a conceptual reactive flow control experiment to delay laminar-to-turbulence transition. Numerical simulations of their flow control system demonstrate almost complete cancellation of single and multiple artificial wave disturbances. Their controller also successfully attenuated a natural disturbance signal with developing wave packets from an actual wind-tunnel experiment.

Jacobson and Reynolds^{48,50,51} used neural networks to minimize the boundary velocity gradient of three model flows: the one-dimensional stochastic Burgers equation, a two-dimensional computational model of the near-wall region of a turbulent boundary layer, and a real-time turbulent flow with a spanwise array of wall actuators together with upstream and downstream wall sensors. For all three problems, the neural network successfully learned about the flow and developed into proficient controllers. For the laboratory experiments, however, Jacobson and Reynolds⁵⁰ report that the neural network training time was much longer and the performance was no better than a simpler ad hoc controller that they developed. Jacobson and Reynolds emphasize that alternative neural net configurations and convergence algorithms may, however, greatly improve the network performance.

Using the angle of attack and angular velocity as inputs, Fallor et al.¹⁵⁵ trained a neural network to model the measured unsteady surface pressure over a pitching airfoil (also see Schreck et al.¹⁵⁶) Following training and using the instantaneous angle of attack and pitch rate as the only inputs, their¹⁵⁵ network was able to predict accurately the surface pressure topology as well as the time-dependent aerodynamic forces and moments. The model was then used to develop a neural network controller for wing-motion actuator signals, which in turn provided direct control of the lift-to-drag ratio across a wide range of time-dependent motion histories.

As a final example, Kawthar-Ali and Acharya¹⁵⁷ developed a neural network controller for use in suppressing the dynamic-stall vortex that periodically develops in the leading edge of a pitching airfoil. Based on the current state of the unsteady pressure field, their control system specified the optimum amount of leading-edge suction to achieve complete vortex suppression.

IX. Use of MEMS for Reactive Control

Current usage for MEMS includes accelerometers for airbags and guidance systems, pressure sensors for engine air intake and blood analysis, rate gyroscopes for antilock brakes, microrelays and microswitches for semiconductor automatic test equipment, and microgrippers for surgical procedures.^{35,36,158–178} There is considerable work under way to include other applications, one example being the micro-steam-engine described by Lipkin,¹⁷⁹ Garcia and Sniegowski,^{180,181} and Sniegowski and Garcia.¹⁸² A second example is the 3×1.5 cm digital light processor that contains $0.5\text{--}2 \times 10^6$ individually addressable micromirrors each typically measuring $16\text{ }\mu\text{m}$ on a side. Texas Instruments, Inc., is currently producing such device with a resolution of 2000×1000 pixel, for high-definition televisions and other display equipments. The company maintains that when mass produced, such device would cost on the order of \$100, that is, less than \$0.0001 per actuator.

MEMS would be ideal for the reactive flow control concept advocated in the present paper. Methods of flow control targeted toward specific coherent structures involve nonintrusive detection and subsequent modulation of events that occur randomly in space and time. To achieve proper targeted control of these quasi-periodic vortical events, temporal phasing as well as spatial selectivity are

required. Practical implementation of such an idea necessitates the use of a large number of intelligent, communicative wall sensors and actuators arranged in a checkerboard pattern. In this paper, we have provided estimates for the number, characteristics and energy consumption of such elements required to modulate the turbulent boundary layer that develops along a typical commercial aircraft or nuclear submarine. An upper-bound number to achieve total turbulence suppression is about 1×10^6 sensors/actuators per square meter of the surface, although as argued earlier the actual number needed to achieve effective control could perhaps be 1–2 orders of magnitude below that.

The sensors would be expected to measure the amplitude, location, and phase or frequency of the signals impressed on the wall by incipient bursting events. Instantaneous wall-pressure or wall-shear stress can be sensed, for example. The normal or in-plane motion of a minute membrane is proportional to the respective point force of primary interest. For measuring wall pressure, microphonelike devices respond to the motion of a vibrating surface membrane or an internal elastomer. Several types are available including variable capacitance (condenser or eletret); ultrasonic; optical, for example, optical fiber and diode laser; and piezoelectric devices (for example, see Refs. 38, 183, and 184). A potentially useful technique for our purposes has been tried at the Massachusetts Institute of Technology.^{185–188} An array of extremely small (0.2 mm in diameter) laser-powered microphones (termed picophones) was machined in silicon, using integrated circuit fabrication techniques, and was used for field measurement of the instantaneous surface pressure in a turbulent boundary layer. The wall-shear stress, though smaller and, therefore, more difficult to measure than pressure, provides a more reliable signature of the near-wall events.

Actuators are expected to produce a desired change in the targeted coherent structures. The local acceleration action needed to stabilize an incipient bursting event can be in the form of adaptive wall, transpiration, wall heat transfer, or electromagnetic body force. Traveling surface waves can be used to modify a locally convecting pressure gradient such that the wall motion follows that of the coherent event causing the pressure change. Surface motion in the form of a Gaussian hill with height $y^+ = O[10]$ should be sufficient to suppress typical incipient bursts.^{81,189} Such time-dependent alteration in wall geometry can be generated by driving a flexible skin using an array of piezoelectric devices (dilate or contract depending on the polarity of current passing through them), electromagnetic actuators, magnetoelastic ribbons (made of nonlinear materials that change their stiffness in the presence of varying magnetic fields), or Terfenol-d rods (a novel metal composite, developed at Grumman Corporation, which changes its length when subjected to a magnetic field). Note should also be made of other exotic materials that can be used for actuation. For example, electrorheological fluids¹⁹⁰ instantly solidify when exposed to an electric field and may, thus, be useful for the present application. Recently constructed microactuators specifically designed for flow control include those by Wiltse and Glezer,¹⁹¹ James et al.,⁵³ Jacobson and Reynolds,⁵⁰ Vargo and Muntz,¹⁹² and Keefe.⁵⁴

Suction/injection at many discrete points can be achieved by simply connecting a large number of minute streamwise slots, arranged in a checkerboard pattern, to a low-pressure/high-pressure reservoir located underneath the working surface. The transpiration through each individual slot is turned on and off using a corresponding number of independently controlled microvalves. Alternatively, positive displacement or rotary micropumps (for example, see Sen et al.¹⁹³ and Sharatchandra et al.¹⁹⁴) can be used for blowing or sucking fluid through small holes/slits. Based on the results of Gad-el-Hak and Blackwelder,⁶⁵ equivalent suction coefficients of about 0.0006 should be sufficient to stabilize the near-wall region. When the skin-friction coefficient in the uncontrolled boundary layer is assumed to be $C_f = 0.003$ and the suction used is assumed to be sufficient to establish an asymptotic boundary layer ($d\delta_0/dx = 0$, where δ_0 is the momentum thickness), the skin friction in the reactively controlled case is then $C_f = 0 + 2 C_q = 0.0012$, or 40% of the original value. The net benefit will, of course, be reduced by the energy expenditure of the suction pump (or

micropumps), as well as the array of microsensors and microvalves.

Finally, if the bursting events are to be eliminated by lowering the near-wall viscosity, direct electric-resistance heating can be used in liquid flows and thermoelectric devices based on the Peltier effect can be used for cooling in the case of gaseous boundary layers. The absolute viscosity of water at 20°C decreases by approximately 2% for each 1°C rise in temperature, whereas for room-temperature air, μ decreases by approximately 0.2% for each 1°C drop in temperature. The streamwise momentum equation written at the wall can be used to show that a suction coefficient of 0.0006 has approximately the same effect on the wall curvature of the instantaneous velocity profile as a surface heating of 2°C in water or a surface cooling of 40°C in air.^{195,196}

Sensors and actuators of the types discussed in this section can be combined on individual electronic chips using microfabrication technology. The chips can be interconnected in a communication network that is controlled by a massively parallel computer or a self-learning neural network, perhaps each sensor/actuator unit communicating only with its immediate neighbors. In other words, it may not be necessary for one sensor/actuator to exchange signals with another far away unit. Factors to be considered in an eventual field application of chips produced using microfabrication processes include sensitivity of sensors, sufficiency and frequency response of actuators' action, fabrication of large arrays at affordable prices, survivability in the hostile field environment, and energy required to power the sensors/actuators. As argued by Gad-el-Hak,^{4,55,56} sensor/actuator chips currently produced are small enough for typical field application, and they can be programmed to provide a sufficiently large/fast action in response to a certain sensor output (see also Jacobson and Reynolds⁵⁰). Present prototypes are, however, still quite expensive as well as delicate. However, so was the transistor when first introduced. It is hoped that the unit price of future sensor/actuator elements would follow the same dramatic trends witnessed in case of the simple transistor and even the much more complex integrated circuit. The price anticipated by Texas Instruments for an array of 1×10^6 mirrors hints that the technology is well in its way to mass produce phenomenally inexpensive microsensors and microactuators. Additionally, current automotive applications are a rigorous proving ground for MEMS: Under-the-hood sensors can already withstand harsh conditions such as intense heat, shock, continual vibration, corrosive gases, and electromagnetic fields.

X. Summary

The frontiers of the field of flow control have been emphasized, and the important advances that have taken place during the past few years and that provide a blueprint for future progress have been reviewed. In two words, the future of flow control is in taming turbulence by targeting its coherent structures: reactive control. Recent developments in chaos control, microfabrication, and soft computing tools are making it more feasible to perform reactive control of turbulent flows to achieve drag reduction, lift enhancement, mixing augmentation, and noise suppression. Field applications, however, have to await further progress in those three modern areas.

The outlook for reactive control is quite optimistic. Soft computing tools and nonlinear dynamical systems theory are developing at fast pace. MEMS technology is improving even faster. The ability of Texas Instruments to produce an array of 1×10^6 individually addressable mirrors for around \$0.01 per actuator is a foreteller of the spectacular advances anticipated in the near future. Existing automotive applications of MEMS have already proven the ability of such devices to withstand the harsh environment under the hood. For the first time, targeted control of turbulent flows is now in the realm of the possible for future practical devices. What is needed now is a focused, well-funded research and development program to make it all come together for field application of reactive flow control systems.

In parting, it may be worth recalling that a mere 10% reduction in the total drag of an aircraft translates into a saving of \$1 billion in annual fuel cost at 1999 prices for the commercial fleet of aircraft in the United States alone. (Perhaps even a bigger saving of \$1.6 billion

at 2000 prices.) Contrast this benefit to the annual worldwide expenditure of perhaps a few million dollars for all basic research in the broad field of flow control. Taming turbulence, though arduous, will pay for itself in gold. Reactive control as difficult as it seems, is neither impossible nor a pie in the sky. Beside, lofty goals require strenuous efforts. Easy solutions to difficult problems are likely to be wrong as implied by the witty words of the famed journalist Henry Louis Mencken (1880–1956): “There is always an easy solution to every human problem—neat, plausible and wrong.”

References

- ¹Prandtl, L., “Über Flüssigkeitsbewegung bei sehr kleiner Reibung,” *Proceedings of the Third International Math. Congress*, 1904, pp. 484–491.
- ²Lachmann, G. V. (ed.), *Boundary Layer and Flow Control*, Vols. 1 and 2, Pergamon, Oxford, England, U.K., 1961.
- ³Gad-el-Hak, M., Pollard, A., and Bonnet, J.-P., *Flow Control: Fundamentals and Practices*, Springer-Verlag, Berlin, 1998.
- ⁴Gad-el-Hak, M., *Flow Control: Passive, Active and Reactive Flow Management*, Cambridge Univ. Press, London, 2000.
- ⁵Fiedler, H. E., and Fernholz, H.-H., “On Management and Control of Turbulent Shear Flows,” *Progress in Aerospace Science*, Vol. 27, 1990, pp. 305–387.
- ⁶Moin, P., and Bewley, T., “Feedback Control of Turbulence,” *Applied Mechanics Reviews*, Vol. 47, No. 6, Pt. 2, 1994, pp. S3–S13.
- ⁷Gutmark, E. J., and Ho, C.-M., “Visualization of a Forced Elliptical Jet,” *AIAA Journal*, Vol. 24, 1986, pp. 684, 685.
- ⁸Williams, D. R., and Amato, C. W., “Unsteady Pulsing of Cylinder Wakes,” *Frontiers in Experimental Fluid Mechanics*, edited by M. Gad-el-Hak, Springer-Verlag, New York, 1989, pp. 337–364.
- ⁹Fiedler, H. E., Glezer, A., and Wygnanski, I., “Control of Plane Mixing Layer: Some Novel Experiments,” *Current Trends in Turbulence Research*, edited by H. Branover, M. Mond, and Y. Unger, AIAA, Washington, DC, 1988, pp. 30–64.
- ¹⁰Gutmark, E. J., Schadow, K. C., and Yu, K. H., “Mixing Enhancement in Supersonic Free Shear Flows,” *Annual Review of Fluid Mechanics*, Vol. 27, 1995, pp. 375–417.
- ¹¹Viswanath, P. R., “Flow Management Techniques for Base and Afterbody Drag Reduction,” *Progress in Aerospace Science*, Vol. 32, 1995, pp. 79–129.
- ¹²Gutmark, E. J., and Grinstein, F. F., “Flow Control with Noncircular Jets,” *Annual Review of Fluid Mechanics*, Vol. 31, 1999, pp. 239–272.
- ¹³Bushnell, D. M., “Turbulent Drag Reduction for External Flows,” AIAA Paper 83-0227, 1983.
- ¹⁴Bushnell, D. M., “Viscous Drag Reduction in Aeronautics,” *Proceedings of the 19th Congress of the International Council of the Aeronautical Sciences*, Vol. 1, AIAA, Washington, DC, 1994, pp. XXXIII–LVI.
- ¹⁵Wilkinson, S. P., Anders, J. B., Lazos, B. S., and Bushnell, D. M., “Turbulent Drag Reduction Research at NASA Langley: Progress and Plans,” *International Journal on Heat and Fluid Flow*, Vol. 9, 1988, pp. 266–277.
- ¹⁶Bushnell, D. M., and McGinley, C. B., “Turbulence Control in Wall Flows,” *Annual Review of Fluid Mechanics*, Vol. 21, 1989, pp. 1–20.
- ¹⁷Gad-el-Hak, M., “Flow Control,” *Applied Mechanics Reviews*, Vol. 42, 1989, pp. 261–293.
- ¹⁸Bushnell, D. M., and Hefner, J. N. (eds.), *Viscous Drag Reduction in Boundary Layers*, AIAA, Washington, DC, 1990.
- ¹⁹Gad-el-Hak, M., and Bushnell, D. M., “Separation Control: Review,” *Journal of Fluids Engineering*, Vol. 113, 1991, pp. 5–30.
- ²⁰Barnwell, R. W., and Hussaini, M. Y. (eds.), *Natural Laminar Flow and Laminar Flow Control*, Springer-Verlag, New York, 1992.
- ²¹Joslin, R. D., Erlebacher, G., and Hussaini, M. Y., “Active Control of Instabilities in Laminar Boundary Layers—Overview and Concept Validation,” *Journal of Fluids Engineering*, Vol. 118, 1996, pp. 494–497.
- ²²Bussmann, K., and Münz, H., “Die Stabilität der laminaren Reibungsschicht mit Absaugung,” *Jahrb. Dtsch. Luftfahrtforschung*, Vol. 1, 1942, pp. 36–39.
- ²³Pretsch, J., “Umschlagbeginn und Absaugung,” *Jahrb. Dtsch. Luftfahrtforschung*, Vol. 1, 1942, pp. 54–71.
- ²⁴Iglisch, R., “Exakte Berechnung der laminaren Reibungsschicht an der längs angeströmten ebenen Platte mit homogener Absaugung,” *Schriften Dtsch. Akad. Luftfahrtforschung B*, Vol. 8, 1994, pp. 1–51.
- ²⁵Ulrich, A., “Theoretische Untersuchungen über die Widerstandersparnis durch Laminarhaltung mit Absaugung,” *Schriften Dtsch. Akad. Luftfahrtforschung B*, Vol. 8, 1944, p. 53.
- ²⁶Eléna, M., “Etude des Champs Dynamiques et Thermiques d’un Ecoulement Turbulent en Conduit avec Aspiration à la Paroi,” Ph.D. Dissertation, Univ. d’Aix-Marseille, Marseille, France, 1975.
- ²⁷Eléna, M., “Suction Effects on Turbulence Statistics in a Heated Pipe Flow,” *Physics of Fluids*, Vol. 27, 1984, pp. 861–866.
- ²⁸Antonia, R. A., Fulachier, L., Krishnamoorthy, L. V., Benabid, T., and Anselmet, F., “Influence of Wall Suction on the Organized Motion in a Turbulent Boundary Layer,” *Journal of Fluid Mechanics*, Vol. 190, 1988, pp. 217–240.
- ²⁹Favre, A., Dumas, R., Verollet, E., and Coantic, M., “Couche Limite Turbulente sur Paroi Poreuse avec Aspiration,” *J. Mécanique*, Vol. 5, 1966, pp. 3–28.
- ³⁰Rotta, J. C., “Control of Turbulent Boundary Layers by Uniform Injection and Suction of Fluid,” *Seventh Congress of the International Council of the Aeronautical Sciences*, 1970.
- ³¹Verollet, E., Fulachier, L., Dumas, R., and Favre, A., “Turbulent Boundary Layer with Suction and Heating to the Wall,” *Heat and Mass Transfer in Boundary Layers*, edited by N. Afgan, Z. Zaric, and P. Anastasijevic, Vol. 1, Pergamon, Oxford, England, U.K., 1972, pp. 157–168.
- ³²Keith, W. L., Hurdis, D. A., and Abraham, B. M., “A Comparison of Turbulent Boundary Layer Wall-Pressure Spectra,” *Journal of Fluids Engineering*, Vol. 114, 1992, pp. 338–347.
- ³³Gad-el-Hak, M., and Bandyopadhyay, P. R., “Reynolds Number Effects in Wall-Bounded Flows,” *Applied Mechanics Reviews*, Vol. 47, 1994, pp. 307–365.
- ³⁴Aronson, D., Johansson, A. V., and Löfdahl, L., “A Shear-Free Turbulent Boundary Layer—Experiments and Modeling,” *Journal of Fluid Mechanics*, Vol. 338, 1997, pp. 363–385.
- ³⁵Ho, C.-M., and Tai, Y.-C., “Review: MEMS and Its Applications for Flow Control,” *Journal of Fluids Engineering*, Vol. 118, 1996, pp. 437–447.
- ³⁶Ho, C.-M., and Tai, Y.-C., “Micro-Electro-Mechanical Systems (MEMS) and Fluid Flows,” *Annual Review of Fluid Mechanics*, Vol. 30, 1998, pp. 579–612.
- ³⁷Löfdahl, L., Kälvesten, E., and Stemme, G., “Small Silicon Pressure Transducers for Space-Time Correlation Measurements in a Flat Plate Boundary Layer,” *Journal of Fluids Engineering*, Vol. 118, 1996, pp. 457–463.
- ³⁸Löfdahl, L., and Gad-el-Hak, M., “MEMS Applications in Turbulence and Flow Control,” *Progress in Aerospace Science*, Vol. 35, 1999, pp. 101–203.
- ³⁹Wilkinson, S. P., “Interactive Wall Turbulence Control,” *Viscous Drag Reduction in Boundary Layers*, edited by D. M. Bushnell and J. N. Hefner, AIAA, Washington, DC, 1990, pp. 479–509.
- ⁴⁰Alshamani, K. M. M., Livesey, J. L., and Edwards, F. J., “Excitation of the Wall Region by Sound in Fully Developed Channel Flow,” *AIAA Journal*, Vol. 20, 1982, pp. 334–339.
- ⁴¹Wilkinson, S. P., and Balasubramanian, R., “Turbulent Burst Control Through Phase-Locked Surface Depressions,” AIAA Paper 85-0536, 1985.
- ⁴²Nosenchuck, D. M., and Lynch, M. K., “The Control of Low-Speed Streak Bursting in Turbulent Spots,” AIAA Paper 85-0535, 1985.
- ⁴³Breuer, K. S., Haritonidis, J. H., and Landahl, M. T., “The Control of Transient Disturbances in a Flat Plate Boundary Layer Through Active Wall Motion,” *Physics of Fluids A*, Vol. 1, 1989, pp. 574–582.
- ⁴⁴Kwong, A., and Dowling, A., “Active Boundary Layer Control in Diffusers,” AIAA Paper 93-3255, 1993.
- ⁴⁵Reynolds, W. C., “Sensors, Actuators, and Strategies for Turbulent Shear-Flow Control,” *Oral Presentation AIAA Third Flow Control Conference*, July 1993.
- ⁴⁶Jacobs, J., James, R., Ratliff, C., and Glazer, A., “Turbulent Jets Induced by Surface Actuators,” AIAA Paper 93-3243, 1993.
- ⁴⁷Jacobson, S. A., and Reynolds, W. C., “Active Control of Boundary Layer Wall Shear Stress Using Self-Learning Neural Networks,” AIAA Paper 93-3272, 1993.
- ⁴⁸Jacobson, S. A., and Reynolds, W. C., “Active Boundary Layer Control Using Flush-Mounted Surface Actuators,” *Bulletin of the American Physical Society*, Vol. 38, 1993, p. 2197.
- ⁴⁹Jacobson, S. A., and Reynolds, W. C., “Active Control of Transition and Drag in Boundary Layers,” *Bulletin of the American Physical Society*, Vol. 38, 1994, p. 1894.
- ⁵⁰Jacobson, S. A., and Reynolds, W. C., “An Experimental Investigation Towards the Active Control of Turbulent Boundary Layers,” Dept. of Mechanical Engineering, Rept. TF-64, Stanford Univ., Stanford, CA, 1995.
- ⁵¹Jacobson, S. A., and Reynolds, W. C., “Active Control of Streamwise Vortices and Streaks in Boundary Layers,” *Journal of Fluid Mechanics*, Vol. 360, 1998, pp. 179–211.
- ⁵²Fan, X., Hofmann, L., and Herbert, T., “Active Flow Control with Neural Networks,” AIAA Paper 93-3273, 1993.
- ⁵³James, R. D., Jacobs, J. W., and Glezer, A., “Experimental Investigation of a Turbulent Jet Produced by an Oscillating Surface Actuator,” *Applied Mechanics Reviews*, Vol. 47, No. 6, Pt. 2, 1994, pp. S127–S1131.
- ⁵⁴Keefe, L. R., “A MEMS-Based Normal Vorticity Actuator for Near-Wall Modification of Turbulent Shear Flows,” *Proceedings of the Workshop on Flow Control: Fundamentals and Practices*, edited by J.-P. Bonnet, M. Gad-el-Hak, and A. Pollard, 1996, pp. 1–21.

- ⁵⁵Gad-el-Hak, M., "Interactive Control of Turbulent Boundary Layers: A Futuristic Overview," *AIAA Journal*, Vol. 32, 1994, pp. 1753-1765.
- ⁵⁶Gad-el-Hak, M., "Modern Developments in Flow Control," *Applied Mechanics Reviews*, Vol. 49, 1996, pp. 365-379.
- ⁵⁷Lumley, J. L., "Control of Turbulence," AIAA Paper 96-0001, 1996.
- ⁵⁸McMichael, J. M., "Progress and Prospects for Active Flow Control Using Microfabricated Electromechanical Systems (MEMS)," AIAA Paper 96-0306, 1996.
- ⁵⁹Mehregany, M., DeAnna, R. G., and Reshotko, E., "Microelectromechanical Systems for Aerodynamics Applications," AIAA Paper 96-0421, 1996.
- ⁶⁰Bushnell, D. M., "Frontiers of the 'Responsibly Imaginable' in Aeronautics" (Dryden Lecture), AIAA Paper 98-0001, 1998.
- ⁶¹Bandyopadhyay, P. R., "Review—Mean Flow in Turbulent Boundary Layers Disturbed to Alter Skin Friction," *Journal of Fluids Engineering*, Vol. 108, 1986, pp. 127-140.
- ⁶²Cantwell, B. J., "Organized Motion in Turbulent Flow," *Annual Review of Fluid Mechanics*, Vol. 13, 1981, pp. 457-515.
- ⁶³Robinson, S. K., "Coherent Motions in the Turbulent Boundary Layer," *Annual Review of Fluid Mechanics*, Vol. 23, 1991, pp. 601-639.
- ⁶⁴Gad-el-Hak, M., and Blackwelder, R. F., "Drag Reduction Method for Turbulent Boundary Layers," AIAA Paper 87-0358, 1987.
- ⁶⁵Gad-el-Hak, M., and Blackwelder, R. F., "Selective Suction for Controlling Bursting Events in a Boundary Layer," *AIAA Journal*, Vol. 27, 1989, pp. 308-314.
- ⁶⁶Blackwelder, R. F., and Gad-el-Hak, M., "Method and Apparatus for Reducing Turbulent Skin Friction," U.S. Patent 4,932,612, 1990.
- ⁶⁷Swearingen, J. D., and Blackwelder, R. F., "Instantaneous Streamwise Velocity Gradients in the Wall Region," *Bulletin of the American Physical Society*, Vol. 29, 1984, p. 1528.
- ⁶⁸Blackwelder, R. F., and Swearingen, J. D., "The Role of Inflectional Velocity Profiles in Wall Bounded Flows," *Near-Wall Turbulence: 1988 Zoran Zaric Memorial Conference*, edited by S. J. Kline and N. H. Afgan, Hemisphere, New York, 1990, pp. 268-288.
- ⁶⁹Gad-el-Hak, M., and Hussain, A. K. M. F., "Coherent Structures in a Turbulent Boundary Layer. Part 1. Generation of 'Artificial' Bursts," *Physics of Fluids*, Vol. 29, 1986, pp. 2124-2139.
- ⁷⁰Johansen, J. B., and Smith, C. R., "The Effects of Cylindrical Surface Modifications on Turbulent Boundary Layers," *AIAA Journal*, Vol. 24, 1986, pp. 1081-1087.
- ⁷¹Wilkinson, S. P., and Lazos, B. S., "Direct Drag and Hot-Wire Measurements on Thin-Element Riblet Arrays," *Turbulence Management and Relaminarization*, edited by H. W. Liepmann and N. Narasimha, Springer-Verlag, New York, 1987, pp. 121-131.
- ⁷²Wilkinson, S. P., "Direct Drag Measurements on Thin-Element Riblets with Suction and Blowing," AIAA Paper 88-3670, 1988.
- ⁷³Choi, H., Moin, P., and Kim, J., "Active Turbulence Control for Drag Reduction in Wall-Bounded Flows," *Journal of Fluid Mechanics*, Vol. 262, 1994, pp. 75-110.
- ⁷⁴Ott, E., Grebogi, C., and Yorke, J. A., "Controlling Chaos," *Physical Review Letters*, Vol. 64, 1990, pp. 1196-1199.
- ⁷⁵Ott, E., Grebogi, C., and Yorke, J. A., "Controlling Chaotic Dynamical Systems," *Chaos: Soviet-American Perspectives on Nonlinear Science*, edited by D. K. Campbell, American Inst. of Physics, New York, 1990, pp. 153-172.
- ⁷⁶Abergel, F., and Temam, R., "On Some Control Problems in Fluid Mechanics," *Theoretical Computational Fluid Dynamics*, Vol. 1, 1990, pp. 303-325.
- ⁷⁷Choi, H., Temam, R., Moin, P., and Kim, J., "Feedback Control for Unsteady Flow and Its Application to the Stochastic Burgers Equation," *Journal of Fluid Mechanics*, Vol. 253, 1993, pp. 509-543.
- ⁷⁸Bewley, T. R., Moin, P., and Temam, R., "Optimal and Robust Approaches for Linear and Nonlinear Regulation Problems in Fluid Mechanics," AIAA Paper 97-1872, 1997.
- ⁷⁹Bewley, T. R., Temam, R., and Ziane, M., "A General Framework for Robust Control in Fluid Mechanics," Center for Turbulence Research, Rept. CTR-Manuscript-169, Stanford Univ., Stanford, CA, 1998.
- ⁸⁰Sritharan, S. S. (ed.), *Optimal Control of Viscous Flow*, Society for Industrial and Applied Mathematics, Philadelphia, 1998.
- ⁸¹Lumley, J. L., "Control of the Wall Region of a Turbulent Boundary Layer," *Turbulence: Structure and Control*, edited by J. M. McMichael, 1991, pp. 61, 62.
- ⁸²Choi, H., Moin, P., and Kim, J., "Turbulent Drag Reduction: Studies of Feedback Control and Flow Over Riblets," Dept. of Mechanical Engineering, Rept. TF-55, Stanford Univ., Stanford, CA, 1992.
- ⁸³Gad-el-Hak, M., "Innovative Control of Turbulent Flows," AIAA Paper 93-3268, 1993.
- ⁸⁴Gad-el-Hak, M., "Frontiers of Flow Control," *Flow Control: Fundamentals and Practices*, edited by M. Gad-el-Hak, A. Pollard, and J.-P. Bonnet, Springer-Verlag, Berlin, 1998, pp. 109-153.
- ⁸⁵Blackwelder, R. F., "Some Notes on Drag Reduction in the Near-Wall Region," *Flow Control: Fundamentals and Practices*, edited by M. Gad-el-Hak, A. Pollard, and J.-P. Bonnet, Springer-Verlag, Berlin, 1998, pp. 155-198.
- ⁸⁶Delville, J., Cordier, L., and Bonnet, J.-P., "Large-Scale-Structure Identification and Control in Turbulent Shear Flows," *Flow Control: Fundamentals and Practices*, edited by M. Gad-el-Hak, A. Pollard, and J.-P. Bonnet, Springer-Verlag, Berlin, 1998, pp. 199-273.
- ⁸⁷Perrier, P., "Multiscale Active Flow Control," *Flow Control: Fundamentals and Practices*, edited by M. Gad-el-Hak, A. Pollard, and J.-P. Bonnet, Springer-Verlag, Berlin, 1998, pp. 275-334.
- ⁸⁸Berkooz, G., Fisher, M., and Psiaki, M., "Estimation and Control of Models of the Turbulent Wall Layer," *Bulletin of the American Physical Society*, Vol. 38, 1993, p. 2197.
- ⁸⁹Wadsworth, D. C., Muntz, E. P., Blackwelder, R. F., and Shifflett, G. R., "Transient Energy Release Pressure Driven Microactuators for Control of Wall-Bounded Turbulent Flows," AIAA Paper 93-3271, 1993.
- ⁹⁰Lindner, J. F., and Ditto, W. L., "Removal, Suppression and Control of Chaos by Nonlinear Design," *Applied Mechanics Review*, Vol. 48, 1995, pp. 795-808.
- ⁹¹Banks, S. P., *Control Systems Engineering*, Prentice-Hall International, Englewood Cliffs, NJ, 1986.
- ⁹²Petersen, I. R., and Savkin, A. V., *Robust Kalman Filtering for Signals and Systems with Large Uncertainties*, Birkhäuser, Boston, 1999.
- ⁹³Aubry, N., Holmes, P., Lumley, J. L., and Stone, E., "The Dynamics of Coherent Structures in the Wall Region of a Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 192, 1988, pp. 115-173.
- ⁹⁴Aubry, N., "Use of Experimental Data for an Efficient Description of Turbulent Flows," *Applied Mechanics Reviews*, Vol. 43, 1990, pp. S240-S245.
- ⁹⁵Pomeau, Y., and Manneville, P., "Intermittent Transition to Turbulence in Dissipative Dynamical Systems," *Communications in Mathematical Physics*, Vol. 74, 1980, pp. 189-197.
- ⁹⁶Berkooz, G., Holmes, P., and Lumley, J. L., "Intermittent Dynamics in Simple Models of the Turbulent Boundary Layer," *Journal of Fluid Mechanics*, Vol. 230, 1991, pp. 75-95.
- ⁹⁷Holmes, P., Lumley, J. L., and Berkooz, G., *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*, Cambridge Univ. Press, Cambridge, England, U.K., 1996.
- ⁹⁸Grappin, R., and Léorat, J., "Lyapunov Exponents and the Dimension of Periodic Incompressible Navier-Stokes Flows: Numerical Measurements," *Journal of Fluid Mechanics*, Vol. 222, 1991, pp. 61-94.
- ⁹⁹Deane, A. E., and Sirovich, L., "A Computational Study of Rayleigh-Bénard Convection. Part 1. Rayleigh-Number Scaling," *Journal of Fluid Mechanics*, Vol. 222, 1991, pp. 231-250.
- ¹⁰⁰Sirovich, L., and Deane, A. E., "A Computational Study of Rayleigh-Bénard Convection. Part 2. Dimension Considerations," *Journal of Fluid Mechanics*, Vol. 222, 1991, pp. 251-265.
- ¹⁰¹Keefe, L. R., Moin, P., and Kim, J., "The Dimension of Attractors Underlying Periodic Turbulent Poiseuille Flow," *Journal of Fluid Mechanics*, Vol. 242, 1992, pp. 1-29.
- ¹⁰²Fowler, T. B., "Application of Stochastic Control Techniques to Chaotic Nonlinear Systems," *IEEE Transactions on Automatic Control*, Vol. 34, 1989, pp. 201-205.
- ¹⁰³Hübner, A., and Lüscher, E., "Resonant Stimulation and Control of Nonlinear Oscillators," *Naturwissenschaften*, Vol. 76, 1989, pp. 67-69.
- ¹⁰⁴Huberman, B., "The Control of Chaos," *Proceedings of the Workshop on Applications of Chaos*.
- ¹⁰⁵Huberman, B. A., and Lumer, E., "Dynamics of Adaptive Systems," *IEEE Transactions on Circuits and Systems*, Vol. 37, 1990, pp. 547-550.
- ¹⁰⁶Shinbrot, T., Ott, E., Grebogi, C., and Yorke, J. A., "Using Chaos to Direct Trajectories to Targets," *Physical Review Letters*, Vol. 65, 1990, pp. 3215-3218.
- ¹⁰⁷Shinbrot, T., Ditto, W., Grebogi, C., Ott, E., Spano, M., and Yorke, J. A., "Using the Sensitive Dependence of Chaos (the 'Butterfly Effect') to Direct Trajectories in an Experimental Chaotic System," *Physical Review Letters*, Vol. 68, 1992, pp. 2863-2866.
- ¹⁰⁸Shinbrot, T., Grebogi, C., Ott, E., and Yorke, J. A., "Using Chaos to Target Stationary States of Flows," *Physics Letters A*, Vol. 169, 1992, pp. 349-354.
- ¹⁰⁹Shinbrot, T., Ott, E., Grebogi, C., and Yorke, J. A., "Using Chaos to Direct Orbits to Targets in Systems Describable by a One-Dimensional Map," *Physical Review A: General Physics*, Vol. 45, 1992, pp. 4165-4168.
- ¹¹⁰Shinbrot, T., Bresler, L., and Ottino, J. M., "Manipulation of Isolated Structures in Experimental Chaotic Fluid Flows," *Experimental Thermal and Fluid Science*, Vol. 16, 1998, pp. 76-83.
- ¹¹¹Romeiras, F. J., Grebogi, C., Ott, E., and Dayawansa, W. P., "Controlling Chaotic Dynamical Systems," *Physica D*, Vol. 58, 1992, pp. 165-192.
- ¹¹²Shinbrot, T., Grebogi, C., Ott, E., and Yorke, J. A., "Using Small Perturbations to Control Chaos," *Nature*, Vol. 363, 1993, pp. 411-417.

- ¹¹³Shinbrot, T., "Chaos: Unpredictable Yet Controllable?" *Nonlinear Science Today*, Vol. 3, 1993, pp. 1-8.
- ¹¹⁴Shinbrot, T., "Progress in the Control of Chaos," *Advances in Physics*, Vol. 44, 1995, pp. 73-111.
- ¹¹⁵Shinbrot, T., "Chaos, Coherence and Control," *Flow Control: Fundamentals and Practices*, edited by M. Gad-el-Hak, A. Pollard, and J.-P. Bonnet, Springer-Verlag, Berlin, 1998, pp. 501-527.
- ¹¹⁶Kostelich, E. J., Grebogi, C., Ott, E., and Yorke, J. A., "Targeting from Time Series," *Bulletin of the American Physical Society*, Vol. 38, 1993, p. 2194.
- ¹¹⁷Ditto, W. L., Raueo, S. N., and Spano, M. L., "Experimental Control of Chaos," *Physical Review Letters*, Vol. 65, 1990, pp. 3211-3214.
- ¹¹⁸Ditto, W. L., and Pecora, L. M., "Mastering Chaos," *Scientific American*, Vol. 269, Aug. 1993, pp. 78-84.
- ¹¹⁹Garfinkel, A., Spano, M. L., Ditto, W. L., and Weiss, J. N., "Controlling Cardiac Chaos," *Science*, Vol. 257, 1992, pp. 1230-1235.
- ¹²⁰Auerbach, D., Grebogi, C., Ott, E., and Yorke, J. A., "Controlling Chaos in High Dimensional Systems," *Physical Review Letters*, Vol. 69, 1992, pp. 3479-3482.
- ¹²¹Kostelich, E. J., Grebogi, C., Ott, E., and Yorke, J. A., "Higher-Dimensional Targeting," *Physical Review E*, Vol. 47, 1993, pp. 305-310.
- ¹²²Lai, Y.-C., Deng, M., and Grebogi, C., "Controlling Hamiltonian Chaos," *Physical Review E*, Vol. 47, 1993, pp. 86-92.
- ¹²³Lai, Y.-C., Tél, T., and Grebogi, C., "Stabilizing Chaotic-Scattering Trajectories Using Control," *Physical Review E*, Vol. 48, 1993, pp. 709-717.
- ¹²⁴Lai, Y.-C., and Grebogi, C., "Synchronization of Chaotic Trajectories Using Control," *Physical Review E*, Vol. 47, 1993, pp. 2357-2360.
- ¹²⁵Hayes, S., Grebogi, C., and Ott, E., "Communicating with Chaos," *Physical Review Letters*, Vol. 70, 1994, pp. 3031-3040.
- ¹²⁶Hayes, S., Grebogi, C., Ott, E., and Mark, A., "Experimental Control of Chaos for Communication," *Physical Review Letters*, Vol. 73, 1994, pp. 1781-1784.
- ¹²⁷Lai, Y.-C., Grebogi, C., and Tél, T., "Controlling Transient Chaos in Dynamical Systems," *Towards the Harnessing of Chaos*, edited by M. Yamaguchi, Elsevier, Amsterdam, 1994.
- ¹²⁸Chen, C.-C., Wolf, E. E., and Chang, H.-C., "Low-Dimensional Spatiotemporal Thermal Dynamics on Nonuniform Catalytic Surfaces," *Journal of Physical Chemistry*, Vol. 97, 1993, pp. 1055-1064.
- ¹²⁹Qin, F., Wolf, E. E., and Chang, H.-C., "Controlling Spatiotemporal Patterns on a Catalytic Wafer," *Physical Review Letters*, Vol. 72, 1994, pp. 1459-1462.
- ¹³⁰Auerbach, D., "Controlling Extended Systems of Chaotic Elements," *Physical Review Letters*, Vol. 72, 1994, pp. 1184-1187.
- ¹³¹Keefe, L. R., "Two Nonlinear Control Schemes Contrasted in a Hydrodynamic Model," *Physics of Fluids A*, Vol. 5, 1993, pp. 931-947.
- ¹³²Keefe, L. R., "Drag Reduction in Channel Flow Using Nonlinear Control," AIAA Paper 93-3279, 1993.
- ¹³³Lüscher, E., and Hübner, A., "Resonant Stimulation of Complex Systems," *Helvetica Physica Acta*, Vol. 62, 1989, pp. 544-551.
- ¹³⁴Singer, J., Wang, Y.-Z., and Bau, H. H., "Controlling a Chaotic System," *Physical Review Letters*, Vol. 66, 1991, pp. 1123-1125.
- ¹³⁵Wang, Y., Singer, J., and Bau, H. H., "Controlling Chaos in a Thermal Convection Loop," *Journal of Fluid Mechanics*, Vol. 237, 1992, pp. 479-498.
- ¹³⁶Tang, J., and Bau, H. H., "Stabilization of the No-Motion State in Rayleigh-Bénard Convection through the Use of Feedback Control," *Physical Review Letters*, Vol. 70, 1993, pp. 1795-1798.
- ¹³⁷Tang, J., and Bau, H. H., "Feedback Control Stabilization of the No-Motion State of a Fluid Confined in a Horizontal Porous Layer Heated from Below," *Journal of Fluid Mechanics*, Vol. 257, 1993, pp. 485-505.
- ¹³⁸Hu, H. H., and Bau, H. H., "Feedback Control to Delay or Advance Linear Loss of Stability in Planar Poiseuille Flow," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 447, 1994, pp. 299-312.
- ¹³⁹Corke, T. C., Glauser, M. N., and Berkooz, G., "Utilizing Low-Dimensional Dynamical Systems Models to Guide Control Experiments," *Applied Mechanics Reviews*, Vol. 47, No. 6, Pt. 2, 1994, pp. S132-S138.
- ¹⁴⁰Coller, B. D., Holmes, P., and Lumley, J. L., "Control of Bursting in Boundary Layer Models," *Applied Mechanics Reviews*, Vol. 47, No. 6, Pt. 2, 1994, pp. S139-S143.
- ¹⁴¹Coller, B. D., Holmes, P., and Lumley, J. L., "Control of Noisy Heteroclinic Cycles," *Physica D*, Vol. 72, 1994, pp. 135-160.
- ¹⁴²Shinbrot, T., and Ottino, J. M., "Geometric Method to Create Coherent Structures in Chaotic Flows," *Physical Review Letters*, Vol. 71, 1993, pp. 843-846.
- ¹⁴³Shinbrot, T., and Ottino, J. M., "Using Horseshoes to Create Coherent Structures in Chaotic Fluid Flows," *Bulletin of the American Physical Society*, Vol. 38, 1993, p. 2194.
- ¹⁴⁴Yager, R. R., Zadeh, L. A. (eds.), *An Introduction to Fuzzy Logic Applications in Intelligent Systems*, Kluwer Academic, Boston, 1992.
- ¹⁴⁵Bouchon-Meunier, B., Yager, R. R., and Zadeh, L. A. (eds.), *Fuzzy Logic and Soft Computing*, World Scientific, Singapore, 1995.
- ¹⁴⁶Bouchon-Meunier, B., Yager, R. R., and Zadeh, L. A. (eds.), *Advances in Intelligent Computing—IPMU'94*, Lecture Notes in Computer Science, Vol. 945, Springer-Verlag, Berlin, 1995.
- ¹⁴⁷Jang, J.-S. R., Sun, C.-T., and Mizutani, E., *Neuro-Fuzzy and Soft Computing*, Prentice-Hall, Upper Saddle River, NJ, 1997.
- ¹⁴⁸Noor, A., and Jorgensen, C. C., "A Hard Look at Soft Computing," *Aerospace America*, Vol. 34, Sept. 1996, pp. 34-39.
- ¹⁴⁹Ouellette, J., "Electronic Noses Sniff Out New Markets," *Industrial Physicist*, Vol. 5, No. 1, 1999, pp. 26-29.
- ¹⁵⁰Goldberg, D. E., *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison Wesley Longman, Reading, MA, 1989.
- ¹⁵¹Davis, L. (ed.), *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, New York, 1991.
- ¹⁵²Holland, J. H., *Adaptation in Natural and Artificial Systems*, MIT Press, Cambridge, MA, 1992.
- ¹⁵³Nelson, M. M., and Illingworth, W. T., *A Practical Guide to Neural Nets*, Addison Wesley Longman, Reading, MA, 1991.
- ¹⁵⁴Antsaklis, P. J., "Control Theory Approach," *Mathematical Approaches to Neural Networks*, edited by J. G. Taylor, Elsevier, Amsterdam, 1993, pp. 1-23.
- ¹⁵⁵Faller, W. E., Schreck, S. J., and Luttgies, M. W., "Real-Time Prediction and Control of Three-Dimensional Unsteady Separated Flow Fields Using Neural Networks," AIAA Paper 94-0532, 1994.
- ¹⁵⁶Schreck, S. J., Faller, W. E., and Luttgies, M. W., "Neural Network Prediction of Three-Dimensional Unsteady Separated Flow Fields," *Journal of Aircraft*, Vol. 32, 1995, pp. 178-185.
- ¹⁵⁷Kawthar-Ali, M. H., and Acharya, M., "Artificial Neural Networks for Suppression of the Dynamic-Stall Vortex over Pitching Airfoils," AIAA Paper 96-0540, 1996.
- ¹⁵⁸Angell, J. B., Terry, S. C., and Barth, P. W., "Silicon Micromechanical Devices," *Faraday Transactions I*, Vol. 68, 1983, pp. 744-748.
- ¹⁵⁹Gabriel, K. J., Jarvis, J., and Trimmer, W. (eds.), *Small Machines, Large Opportunities: A Report on the Emerging Field of Microdynamics*, National Science Foundation, AT&T Bell Lab., Murray Hill, NJ, 1988.
- ¹⁶⁰Gabriel, K. J., Tabata, O., Shimaoka, K., Sugiyama, S., and Fujita, H., "Surface-Normal Electrostatic/Pneumatic Actuator," *Proceedings of IEEE Micro Electro Mechanical Systems '92*, Inst. of Electrical and Electronics Engineers, New York, 1992, pp. 128-131.
- ¹⁶¹O'Connor, L., "MEMS: Micromechanical Systems," *Mechanical Engineering*, Vol. 114, Feb. 1992, pp. 40-47.
- ¹⁶²Gravesen, P., Branebjerg, J., and Jensen, O. S., "Microfluidics—A Review," *Journal of Microelectromech. Microeng.*, Vol. 3, 1993, pp. 168-182.
- ¹⁶³Bryzek, J., Peterson, K., and McCulley, W., "Micromachines on the March," *IEEE Spectrum*, Vol. 31, May 1994, pp. 20-31.
- ¹⁶⁴Gabriel, K. J., "Engineering Microscopic Machines," *Scientific American*, Vol. 260, Sept. 1995, pp. 150-153.
- ¹⁶⁵Ashley, S., "Getting a Microgrip in the Operating Room," *Mechanical Engineering*, Vol. 118, Sept. 1996, pp. 91-93.
- ¹⁶⁶Hogan, H., "Invasion of the Micromachines," *New Scientist*, Vol. 29, June 1996, pp. 28-33.
- ¹⁶⁷Ouellette, J., "MEMS: Mega Promise for Micro Devices," *Mechanical Engineering*, Vol. 118, Oct. 1996, pp. 64-68.
- ¹⁶⁸Paula, G., "MEMS Sensors Branch Out," *Aerospace America*, Vol. 34, Sept. 1996, pp. 26-32.
- ¹⁶⁹Robinson, E. Y., Helvajian, H., and Jansen, S. W., "Small and Smaller: The World of MNT," *Aerospace America*, Vol. 34, Sept. 1996, pp. 26-32.
- ¹⁷⁰Robinson, E. Y., Helvajian, H., and Jansen, S. W., "Big Benefits from Tiny Technologies," *Aerospace America*, Vol. 34, Oct. 1996, pp. 38-43.
- ¹⁷¹Madou, M., *Fundamentals of Microfabrication*, CRC Press, Boca Raton, FL, 1997.
- ¹⁷²Tien, N. C., "Silicon Micromachined Thermal Sensors and Actuators," *Microscale Thermophys. Eng.*, Vol. 1, 1997, pp. 275-292.
- ¹⁷³Amato, I., "Forming a Revolution in Miniature," *Science*, Vol. 282, No. 5388, 16 Oct. 1998, pp. 402-405.
- ¹⁷⁴Busch-Vishniac, I. J., "Trends in Electromechanical Transduction," *Physics Today*, Vol. 51, July 1998, pp. 28-34.
- ¹⁷⁵Kovacs, G. T. A., *Micromachined Transducers Sourcebook*, McGraw-Hill, New York, 1998.
- ¹⁷⁶Knight, J., "Dust Mite's Dilemma," *New Scientist*, Vol. 162, No. 2180, 29 May 1999, pp. 40-43.
- ¹⁷⁷Epstein, A. H., "The Inevitability of Small," *Aerospace America*, Vol. 38, March 2000, pp. 30-37.
- ¹⁷⁸O'Connor, L., and Hutchinson, H., "Skyscrapers in a Microworld," *Mechanical Engineering*, Vol. 122, March 2000, pp. 64-67.
- ¹⁷⁹Lipkin, R., "Micro Steam Engine Makes Forceful Debut," *Science News*, Vol. 144, Sept. 1993, p. 197.

- ¹⁸⁰Garcia, E. J., and Sniegowski, J. J., "The Design and Modelling of a Comb-Drive-Based Microengine for Mechanism Drive Applications," *Proceeding of Seventh International Conference on Solid-State Sensors and Actuators (Transducers '93)*, 1993, pp. 763-766.
- ¹⁸¹Garcia, E. J., and Sniegowski, J. J., "Surface Micromachined Micro-engine," *Sensors and Actuators A*, Vol. 48, 1995, pp. 203-214.
- ¹⁸²Sniegowski, J. J., and Garcia, E. J., "Surface Micromachined Gear Trains Driven by an On-Chip Electrostatic Microengine," *IEEE Electron Device Letters*, Vol. 17, July 1996, p. 366.
- ¹⁸³Löfdahl, L., Glavmo, M., Johansson, B., and Stemme, G., "A Silicon Transducer for the Determination of Wall-Pressure Fluctuations in Turbulent Boundary Layers," *Applied Scientific Res.*, Vol. 51, 1993, pp. 203-207.
- ¹⁸⁴Löfdahl, L., Kälvesten, E., and Stemme, G., "Small Silicon Based Pressure Transducers for Measurements in Turbulent Boundary Layers," *Experiments in Fluids*, Vol. 17, 1994, pp. 24-31.
- ¹⁸⁵Warkentin, D. J., Haritonidis, J. H., Mehregany, M., and Senturia, S. D., "A Micromachined Microphone with Optical Interference Readout," *Proceedings of the Fourth International Conference on Solid-State Sensors and Actuators (Transducers '87)*, 1987.
- ¹⁸⁶Young, A. M., Goldsberry, J. E., Haritonidis, J. H., Smith, R. I., and Senturia, S. D., "A Twin-Interferometer Fiber-Optic Readout for Diaphragm Pressure Transducers," *IEEE Solid-State Sensor and Actuator Workshop*, Inst. of Electrical and Electronics Engineers, New York, 1988.
- ¹⁸⁷Haritonidis, J. H., Senturia, S. D., Warkentin, D. J., and Mehregany, M., "Optical Micropressure Transducer," U.S. Patent 4,926,696, 1990.
- ¹⁸⁸Haritonidis, J. H., Senturia, S. D., Warkentin, D. J., and Mehregany, M., "Pressure Transducer Apparatus," U.S. Patent 4,942,767, 1990.
- ¹⁸⁹Carlson, H. A., and Lumley, J. L., "Flow Over an Obstacle Emerging from the Wall of a Channel," *AIAA Journal*, Vol. 34, 1996, pp. 924-931.
- ¹⁹⁰Halsey, T. C., and Martin, J. E., "Electrorheological Fluids," *Scientific American*, Vol. 269, Oct. 1993, pp. 58-64.
- ¹⁹¹Wiltse, J. M., and Glezer, A., "Manipulation of Free Shear Flows Using Piezoelectric Actuators," *Journal of Fluid Mechanics*, Vol. 249, 1993, pp. 261-285.
- ¹⁹²Vargo, S. E., and Muntz, E. P., "Simple Micromechanical Compressor and Vacuum Pump for Flow Control and Other Distributed Applications," AIAA Paper 96-0310, 1996.
- ¹⁹³Sen, M., Wajerski, D., and Gad-el-Hak, M., "A Novel Pump for MEMS Applications," *Journal of Fluids Engineering*, Vol. 118, 1996, pp. 624-627.
- ¹⁹⁴Sharatchandra, M. C., Sen, M., and Gad-el-Hak, M., "Navier-Stokes Simulations of a Novel Viscous Pump," *Journal of Fluids Engineering*, Vol. 119, 1997, pp. 372-382.
- ¹⁹⁵Liepmann, H. W., and Nosenchuck, D. M., "Active Control of Laminar-Turbulent Transition," *Journal of Fluid Mechanics*, Vol. 118, 1982, pp. 201-204.
- ¹⁹⁶Liepmann, H. W., Brown, G. L., and Nosenchuck, D. M., "Control of Laminar Instability Waves Using a New Technique," *Journal of Fluid Mechanics*, Vol. 118, 1982, pp. 187-200.